VOL.
5
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VOL.
5
1954

OPERATIONAL RESEARCH QUARTERLY

VOL. 5 NO. 1 MARCH 1954

Contents

O.R. and Accounting——STAFFORD BEER 1
Conferences in the U.S.A. 12
Capacity; Its Meaning, Use and Measurement . . 13
Capacity of Roads——J. G. WARDROP 14
Capacities in a Manufacturing Process——R. SOLT . 24
The Flow of Motor Spirit from Refinery to Consumer S. SHRAGAH 26
Abstracts and Reviews 33

VOL. 5 NO. 2 JUNE 1954

Contents

Supply and Demand 37
Reflections on Operational Research——E. C. WILLIAMS . . . 39
A Survey of Operations Research in the U.S.A.——ELLIS JOHNSON . 43
Abstracts and Reviews 48
Road Research 50
Large Number Research——MANFRED KNAYER 54
Recent Courses 56

VOL. 5 NO. 3 SEPTEMBER 1954

Contents

Who plans what? 57
Inspection——E. D. VAN REST 60
Assessing Priorities for Technical Effort——G. H. L. ANDREW . 67
Criminology and Prediction——L. T. WILKINS 80
Abstracts and Reviews 87
O. R. and Accounting 92

VOL
5
195

VOL.
5
1954

VOL. 5 NO. 4 DECEMBER 1954

Contents

Operational Research Society 93
Application of Birth and Death Process to Provision of Spare Machines
——J. TAYLOR and R. R. P. JACKSON 95
Management in the U.S.A. 108
Queueing Systems with Phase Type Service——R. R. P. JACKSON . . . 109
Army O.R. in the Far East, 1952/54——K. PENNYCUICK 120
Abstracts and Reviews 130

VOL
5
195

VOL.
5
1954

O·R

OPERATIONAL RESEARCH QUARTERLY

VOL. 5 NO. 1 MARCH 1954

Contents

O.R. and Accounting——STAFFORD BEER . . . 1

Conferences in the U.S.A. 12

Capacity; Its Meaning, Use and Measurement . . 13

Capacity of Roads——J. G. WARDROP . . . 14

Capacities in a Manufacturing Process——R. SOLT . 24

The Flow of Motor Spirit from Refinery to Consumer S. SHRAGAH 26

Abstracts and Reviews 33

OPERATIONAL RESEARCH AND ACCOUNTING

by

STAFFORD BEER*

WHEN he heard that I was to give an address on the relationship between operational research and accounting, a friend asked incredulously: "You mean there *is* one?" This very surprise is my justification for speaking to-night. Various rather passive relations could be indicated between operational research and accounting, which arise from a mutual interest in such matters as analytic method, communications, the recording of data, and the use of calculating and sorting machinery. But I shall restrict these remarks to the relation of active influence, first considering in turn the influence exerted by the one on the other. Later both activities will be shown in their functional context, and their interacting influence on each other will be considered in relation to their joint tasks within the organization of which each is a part.

* Samuel Fox and Co. Ltd. This paper is slightly abridged from that read to the O.R. Society on 18 January.

The Influence of Accounting on O.R.

The influence exerted by accounting on operational research seems to be largely indirect, and arises from the fact that accounting has established its own ascendancy in the field of quantitative reporting to management. There appear to be three main causes for this ascendancy. Firstly, accounting is prescribed by law. Secondly, it deals in black and red money: a terminology which has very real meaning for the business executive. Thirdly, the accountant has been in the field of quantitative reporting longer, and is better equipped with aids, than anyone else.

It is inevitable that an ascendant discipline should impose its own criteria on the thought of other people. Accountants rightly specify the data they require—the form the information takes, the extent to which it is detailed, and so on—and insist on their professional notion of what is proper in the form of their reports. The result is that everyone finds it extremely difficult to consider any problem independently of accounting conventions.

Nothing offensive is intended by the use of the word "convention". All language is conventional; were it not, it could support no fixed interpretation. Coventions are not necessarily meaningless, but they are necessarily restraints on a full appreciation of whatever is the case. The corollary asks that sparing use be made of the phrase: "that is a fact". Any report to management—the balance sheet, the statistical report, the works study—is based on an acquaintance with facts; but none is itself a fact.

This is, I think, the theoretical basis of the influence which I will now try to illustrate from different accounting situations.

(*a*) "*On-cost*" *Generalizations*

It is sometimes claimed, either by or for cost accounting, that cost analyses can be prepared to any degree of detail which may be required. This is theoretically the case, but it is surely unrealistic in practice. In normal practice there is always likely to be an element of generalized on-cost to account for overheads. This follows from the limiting amount of works' information which can be either recorded or stored.

Consider an orthodox accounting system operating in a shop throughout which the overheads have been averaged; i.e., the cost of having a job done on any machine in the shop is given by the labour cost plus a fixed percentage on-cost. Any piece of operational research will then appear to affect the total process cost, only as it affects labour cost.

VOL.
5
1954

This situation is fairly satisfactory when the on-cost is directly related to the operating time, and is fairly low in relation to raw material cost. But the on-cost very often incorporates allowances for such remote expenses as publicity, and can also be set at a very high percentage rate. (I have seen on-costs running at over 1,000 per cent). In such circumstances, the accounting measure of the effect of a research proposal will be a poor one. All this affects O.R. in two ways. It makes a true assessment of the effect of any proposal difficult to obtain. Secondly, it ties the apparent effect to labour costs by an unrealistic equation. If the knowledge of this relationship were also to influence its recommendations, operational research would have attained a most regrettable sophistication.

These obvious influences, which are the less dangerous because they are overt, do not however exhaust the problem. Let us now suppose that the shop which we have been considering manufactures products A and B. Product A is rigorously controlled, complicated, and valuable. Product B is less demanding, simple, and relatively cheap. The higher quality of A is attained by the use, not of more labour, but of better control equipment and method.

In this picture, we have set up conditions in which it apparently costs the same to process A and B products. A is intrinsically more valuable, and has a higher raw material value. But it has a much higher final value, and it is evident that part of the cost of its appreciation has been borne by product B. I believe this position to arise very often, often in a less obtrusive and perhaps quite unrecognized way.

Suppose, now, that operational research into sheer productivity calls for extension of the plant devoted to product B, rather than to product A. It is product A, however, which is the more profitable according to its profit margin over alleged cost. Hence proponents of plant modifications for product A find it easy, using no other argument than the alleged profitability, to attract available capital allocations to the wrong plant. When the modifications are complete, and labour per unit product A has been reduced still further, its alleged profitability increases yet again. Thus it becomes less and less likely that the rightful claims of product B plant to capital resources will be recognized; and it becomes more and more likely that product A, far from exploiting the highly lucrative business it appears to command, is becoming genuinely uneconomic.

(*b*) *Particularized Product Costs*

The whole difficulty could be evaded by avoiding the generalizations which cause the trouble. One approach aimed at the elimination

of ignorance about individual products is made by standard costing. But a firm of any complexity will never be able to produce either an historical product cost or a standard product cost which is exact. Once again, conventions must be invoked. It follows that the more determined we are specifically to allocate the overheads, the bigger the O.R. problem of measurement and apportioning, and the more voluminous the costing report. As a result the grouping of products becomes essential at the stage of presentation.

The way in which the groups are formulated will be governed by managerial convenience of classification into types of product, delivery area, or the like, and not by any sort of accounting homogeneity. In particular, I think the structure of the sales organization is very likely to affect these groupings. From the point of view, therefore, of operational research projects such as we have been considering, it may be just as difficult as it was before to win people from what is really an arbitrary set of categories which continues to conceal the heterogeneous operational situation.

There is a further, and possibly quite profound, influence which the standard type of costing may exert on operational matters. Insofar as standard budgets are made to be met rather than improved upon, there is a general implication in this form of costing that no manufacturing profit or loss will be made. That is the pure concept. The company's overall profit margin on any product (the difference between its specific cost and its net realized price) is therefore concentrated in a trading profit or loss at the point of sale.

Now it very often happens that the sales organization is divided into horizontal strata: that is, its sections deal with groups of sales whose common factor is the amount of work put into their manufacture. It is less common to find vertical strata in a sales organization, in which a section deals with all sales of a particular product regardless of the stage of manufacture at which it is sold. This is natural enough, since the markets served themselves tend to divide in this fashion.

As a result of this concentration of profit on the product as sold and the horizontal stratification of the sales organization, it is inevitable that the total margin earned by the whole firm should become associated with end-product manufacturing departments. There can be no recognition, then, of the opportunity cost of the foregone sale of a semi-finished product. Doubtless this factor could be estimated from market research information, but the psychological climate is bad for producing an O.R. report entailing its use.

The more profound implications which I suggested might exist are highly problematical, and would require a long theoretical

OL.
5
954

treatise to evaluate. I therefore incorporate them, without prejudice, in a leading question. Consider three main forms of these accounting conventions: (a) arbitrary product groupings, (b) the extent of the centralization of cost accountability, and (c) the association of total profit margins with the point of sale. To what degree is it possible that these conventions actually influence the whole organization and managerial structure of a company?

(*c*) *Influence on New Thought*

I cannot leave the question of the influence exercised by accounting on operational research without a reference to an hypothesis which would be of immense importance should it prove correct.

If the productivity studies which everyone is making convey the truth, British industry is in a serious position. I suggest that the complete rethinking of industrial affairs is becoming vital. Already developments in automation are exciting comment: the Russians are said to be operating a number of automatic factories; the Rockford Ordnance Plant in America covers 1,000,000 square feet of floor space, and employs in all one hundred and forty workers. Here then is one possible development in which operational research undoubtedly should play a leading role. Its potential importance will, however, be conditioned by the accounting version of its activities. So far we have found that accounting response can effectively obscure the differentials formulated by operational research with a comprehensive on-cost of 1,000 per cent. There is little hope for our differentials in a fully automatic factory, if they have to face the extrapolation of this trend in an on-cost of 100,000 per cent.

If the answer be that automation is quite well understood, and that accountancy can deal with such problems after all, there are other thoughts which may well be less readily embraced. My personal hypothesis is that there is something to be learned from the prodigality of nature; this hypothesis may not be valuable, but we might reasonably wish to assess its value.

It is supposed that the traditional approaches to economy by machine and material conservation, and to quality by slowly lavished care, have become uneconomic. Instead there should be prodigal production on every machine, at perhaps three times orthodox rates. Expensive and deliberative planning and progressing controls, intermediate inspection and rectification should be abandoned, in favour of a generalized statistical control of process variability alone, the final quality of the product being guaranteed to any desired standard by final selective inspection. On the basis

that there is a given statistical chance of success in a prodigal wastage a given return on capital could be predetermined.

This analogy can be pursued, and I find it interesting. The present point, however, is simply this. Revolutionary thought of this type could not carry out its operational research without a revolution in accountancy. For the attitude of financial accounting to inventory and depreciation, and the attitude of cost accounting to the averaging of cost over each piece produced, between them beg the whole question in advance.

Even to-day a product in its finishing stages has become so valuable by present costing methods that the machine yield of good material is probably at least ten times as important as individual machine output. In view of the work which has been invested in the nearly finished product, this interpretation appeals to common sense as well as to cost accountancy; and productivity will certainly not be increased by the frenetic manufacture of rejects. Nonetheless, the present approach seems to depress the pitch of industrial activity in favour of safeguarding good yield at almost any cost. Also, it is in my opinion basically unsound. There must be an economic "break-even" point somewhere between the importance of good yield and output, and I suggest it is not where costing fixes it now.

This present fix is given by the even spread of cost over every unit of product, and this method assumes the erroneous logical premise that the plant is literally ideal for its job, and therefore could produce 100 per cent good work if so minded. On the contrary, success in manufacture is not deductively certain, but inductively precarious, having a statistically calculable probability in every given case.

The proposal in my mind is that every machine might be given an allocation of material inductively liable to be spoilt, which would be recorded as costing precisely nothing. This allocation could be fixed by reference to the inherent variability of the process, measured by operational research, compared with the permissible variability of the product, provided by its specification. This would establish a more productive relationship between the good "yield" and the "sheer output" elements in productivity; furthermore, the structure of costing would then conform to the logical nature of manufacture.

The Influence of O.R. on Accounting

I have already submitted the view that accountancy has established its own practical ascendancy, but the very nature of operational research gives it a theoretical ascendancy as to both method and insight over what is simply an accounting record. It must have been noticeable, for example, during the earlier part of

VOL.
5
1954

this discussion that the accounting limitations which were causing trouble to O.R. could themselves have been largely removed by O.R. techniques. But there does not appear to be any wide recognition of this potentiality. Sir Charles Goodeve has himself propagated the notion of cost analysis by finding the cost component of each resource as the product of price-per-unit-consumed and the rate of consumption. And a good deal of study has been given to consumption rates in several industries. However, we are still on the threshold of any such general application.

The influence now under examination therefore tends to be potential rather than actual: it seems that the full usefulness of O.R. in this field has yet to be demonstrated. In the reports of Anglo-American productivity teams it is surely reasonable to expect accounts of the latest methods in use. Thus it must be indicative of lack of progress when the *Report on Management Accounting* contains only one reference to a distinctively O.R. method—and that hidden in ten paragraphs on routine statistical quality control.

(*a*) *Establishing Accounting Standards*

It may therefore be worth while to begin with a brief case history of an actual application. The United Steel Companies have been among the pioneers in the field of standard costing, being the originators of one particular form; its accountants have made tremendous efforts to provide a standard cost for every product at every stage of manufacture. This is no light undertaking in a steel works, where the almost infinite variety of qualities, conditions, finishes, sizes and tolerances involves the compilation of a vast library of standards.

It must be made clear that a standard performance for these purposes is not a maximum performance but a normal one. The accountant has two orthodox channels of information about what constitutes normality: (a) production records, (b) the advice of production management, which will agree the accounting standards with him. In the particular departmental case of which I am thinking something like 120,000 machine/job standards had to be established.

The *Report on Management Accounting* has something relevant to say about the latest American practice in this same situation. It says (section 7, paragraph 25):

> "In the steelmaking companies all the products of one department are not costed separately because of the immense variety of quality and type involved, but the actual expenditure is charged to groups of similar products, and these group product costs form the basis of inventory valuations and analytical profit and loss statements."

In the project described, however, the Company's accountants were determined not to acquiesce in such defeatism. The task was nevertheless formidable, and the long job of defining normality in every case by consultation and arithmetic was begun. Meanwhile, we had been installing a highly evolved system of production control based on operational research, and at this stage undertook to switch our techniques onto this new task in the service of accountancy. Apart from the saving of time and effort which would result, we felt there were two definite theoretical advantages to be gained. Hitherto, normality had been defined as an undistributed average of experience. For the first time it would be possible to invoke the concept of normality as a statistical distribution. Secondly, it would now be possible for the first time to propose a statistically validated standard for a new product about which no actual production experience had been gathered.

Comprehensive mathematical models of production had already been devised, and so had schedules of optimum machine speeds, operating methods, and handling times. By feeding the quantitative data of a product into the model together with the appropriate optima of performance, it was possible to obtain (from a desk calculator) an objective performance for that product. This objective performance is essentially a target figure, and by no means suitable for standard accounting purposes.

Now a measure of productivity had already been instituted, in the form of a ratio for every job done between the objective performance based on schedules of optima and the actual performance logged. A very large number of these productivity indices had been calculated, to constitute a population of job productivity over a long period. This population had been analysed and standardized statistically. Thus there were already in existence more than fifty productivity groups, each statistically defined as a normally distributed mean productivity of known range and standard deviation. Since the causes of variation had been assigned by significance testing and analysis of variance, it was possible to nominate the appropriate productivity group for every machine-product, whether the job had been done before or not.

It would not be fitting on this occasion to enter into a lengthy dissertation on the statistical techniques and refinements employed, for these were legion. Suffice it to say that this scheme of description was already in force, and kept under constant surveillance by control chart procedures.

The means used to provide the normal performance data required for accounting standards was therefore simple. It was

VOL.
5
1954

necessary merely to use the mathematical models to calculate objective performance, and to weight this figure with the appropriate productivity group norm. All this has been done quickly from our own office, without any further interference with the shop-floor.

It is clear that, in addition to the standard performance itself, we could also have provided: (a) the limits within which that performance could be expected to vary under a given criterion of probability, and (b) a rapid amendment to the standard once a significant change were demonstrated by sampling on the control chart. Both these facilities are used to the full in the production control system, and also in a novel form of report to the management. But so far the cost accountants do not appear to feel that these extra features are of value to their own work.

(*b*) *Cost Groupings*

With the technique just described still in mind, it will be convenient to recapitulate and offer comment on a matter which has constantly recurred in this examination. That is the problem of cost groupings, which I consider to be probably the major single question at issue.

These groups first appeared in an arbitrary form in the systems using on-cost generalizations. The next stage has been to cost every job separately, and to avoid altogether the deadening effect of a group. I have tried to show, however, that although this practice overcomes the immediate objections to costing a group as such, the basic problem survives. The ubiquitous group returns, this time subsequent to costing, and is once again to be found begging questions and exerting the influence of its own composition.

It seems that for one reason or another, notably perhaps for the ultimate reason that policy makers just cannot assimilate thousands of detailed results into either their own minds or their managerial decisions, cost groups must be a feature of accountancy. If this be so, and if each form of grouping so far noticed be objectionable as I contend, then it rests with operational research to find a proper basis for cost group construction.

The basic measure of what is happening in the process must be, I submit, the actual level of output as compared with an ultimate level of output for the same job. The great advantage of this ideal measure is that it can be taken everywhere with commensurable results. This is precisely the productivity index of which I have spoken, and it has been shown that these indices can be grouped with statistical validity and empirical success. The proposal which operational research might make, then, is that these productivity

groups delineate the categories into which costing data should be divided.

If anyone will agree that the method I have used to establish standard performance is satisfactory, then they are surely committed to this new proposal. For under it the standard cost of fifty products in the same productivity group would be given as the cost of the weighted sum of fifty objective performances. This yields more quickly the same result as my present method, which costs the raw sum of fifty weighted performances. The new proposal simply takes the common productivity weighting "outside the bracket", while the detailed behaviour of each of the fifty products is available from the the production control scheme itself.

(*c*) *Opportunity Costs and Budgetary Control*

The third main area in which operational research might influence accounting is that of opportunity costs. I have no practical experience here and will do no more than indicate a line of thought.

In many firms, and not necessarily those engaged in manufacture alone, the possibility of market realization of any product occurs at many stages. As long as all these opportunities are kept open, a state of maximum flexibility exists. When a sale is foregone in favour of further processing, an opportunity cost is incurred. Hence for any momentary situation in the pattern of production a particular level of flexibility exists, and for this level of flexibility the firm is paying a particular price which can be regarded as an overall opportunity cost. Managements, very sensibly, strive after flexibility, which minimizes their risks. But I think it is increasingly vital to-day that firms should appreciate that flexibility is not chosen simply as a matter of preference against rigidity, but is instead a very expensive commodity.

The difficulty in assessing its cost is that the overall opportunity cost which one wishes to find is not the sum of individual opportunity costs. This is because the acceptance or rejection of one opportunity is not an independent decision, but one which alters the remaining possibilities. If there is an opportunity of sale at the alpha, beta, and gamma stages, and an alpha sale is effected, the beta and gamma opportunities disappear.

The permutations and combinations of stages and products in a big firm generate an extremely tangled system of opportunities. The vital need is for a continuous time series of solutions which can be used in the budgetary process. This means that the whole intricate static model has to be made dynamic.

VOL.
5
1954

There is, as far as I know, only one hope of dealing with this immensely difficult costing problem, and that is by operational research through the theory of games and linear programming. Sooner or later this attack will have to be launched. Meanwhile there can be little doubt that most people will say: "Industry has managed very nicely so far without all this sophisticated pretension". The proper reply to this is, I think, that we cannot be sure until the test is made, and that perhaps a discovery in industrial method essential to the recovery of our national prosperity is latent in this possibly academic problem.

The Interaction of O.R. and Accounting

In conclusion, it seems necessary to say that definitions of operational research and accounting indicate a remarkable identity of purpose. Each is concerned with the quantitative presentation of data, and each expects to influence managerial decisions and policy making. Hence it would seem advantageous to each to exploit its relationship with the other.

There are several fields in which mutual consultation would be highly profitable in connection with the relationship between these two services and those whom they serve, There is the question, recently mentioned at this Society, of the difference between managerial curiosity and need, and the cost of meeting each. There are the questions of operational research into the uses made of accounting the information, and accounting examination of the cost of operational research.

I will, however, limit myself to one major point in this connection. It is that the two disciplines may disagree in the reports they make. The best example I can give of this arises when the performance of plant is being discussed. A modern system of standard costs is accounting to a predetermined value representing normality. The deviations it measures are the only source of cost information about the plant's efficiency. If a manufacturing profit is being made, then everything appears to be highly satisfactory.

Operational research may meantime have revealed a different state of affairs, a state of low productivity in which nothing is satisfactory. In an ideal economy, this would never happen. The manufacturing profit might still appear, but a large trading loss would occur because the costing standard established on low productivity would result in an uncompetitive market price.

It is possible that we rely too much on the assumption of an ideal economy. Supposing, as there is reason to suppose, that a certain product is manufactured with low productivity on a national

scale. Every producer will then make an orthodox manufacturing profit, and none will make a trading loss. This I submit as a general industrial ailment, and a dangerous one. There is no channel by which management can ever learn through accountancy that its plant should be brought to a higher pitch of efficiency; on the contrary, there is implicit accounting support for the extension of plant to work on the same basis as before. The company's economy will not immediately suffer as a result, but the national economy ultimately will.

If operational research can provide the other side of this picture, it is performing a great service. The interesting fact is that the true state of affairs about which management wishes to be informed is here enshrined precisely in the disagreement between its two services of O.R. and accountancy.

Many truths are locked in paradox, and it is with this one that I shall stop.

ACKNOWLEDGEMENTS

I wish to thank the Management of Messrs. Samuel Fox & Co. Ltd. for permission to present this paper; and also to express my gratitude to T. H. Umpleby, who (while not in any way accountable for the opinions expressed here) has been the source of information and stimulation on accounting matters over several years.

CONFERENCES IN THE U.S.A.

THE KEEN interest being taken in O.R. in North America was demonstrated at two conferences held in January 1954: the first on *Production and Inventory Control* at the Case Institute of Technology, the second on *Operations Research* organized by the Society for the Advancement of Management in New York. More than 200 attended each conference, and a wide variety of industries was represented from 26 different States and from Canada.

At the Case conference speakers on each of the three mornings gave interesting examples of research into practical problems, particularly applications of linear programming. Afternoons were devoted to informal discussions of subjects relevant to the morning talks.

The S.A.M. conference opened with two general papers on O.R. in the United States and in Great Britain and continued with more detailed examples of accomplishments of O.R. on production, financial, marketing, and inventory problems. There were also sessions on O.R. techniques which included experimental design, waiting problems, linear programming, symbolic logic, decision theory, and information handling.

R. T. E.

VOL.
5
1954

CAPACITY; ITS MEANING, USE AND MEASUREMENT

Three papers were read and discussed on 19 *February, at a joint "open" meeting between the British Institute of Management and the Operational Research Society. Abridgments of the papers are given below, with a summary of the discussion.*

In his opening remarks the Chairman (Sir CHARLES GOODEVE) pointed out that the process of balancing an operating system (such as a factory or a road) consisting of a series of operations in sequence involved balancing the capacities of each part. Capacities were commonly required to be higher than the average to cope with short period fluctuations or peaks of demand.

The word "capacity" was used in three different yet related senses. The most important of these was the "rate capacity" or the ability to do things in a certain time, which term it was often necessary to qualify, as by "lowest cost", "maximum profit" or "maximum" capacity. Rate capacities had to be related to the nature of the input and output Once the conditions and objectives were specified it was usually possible to measure the rate capacity with reasonable accuracy.

The second principal type was "holding capacity" of which there were two sub types, "static" and "pipeline", the amount of material actually in transit or undergoing a process. Materials contained in holding capacities were unproductive and required expensive working capital.

The third type of capacity appeared very prominently in physics, but rarely, so far, in operations. This was the "elastic capacity" related to the extent to which a given system reacts to an alteration in some "intensive" property. Such elastic capacities were found in economic systems, but so far it had not been possible to measure them very precisely.

Rate capacities and holding capacities might themselves have an elastic component. For example, the maximum profit capacity of a factory was dependent on the price of its product, among other things. The factory could be considered as having a basic economic capacity for standard conditions plus a variable component. Defining and measuring capacities for steady state conditions presented few difficulties but most operating systems involved fluctuations of one sort or another. These fluctuations might arise from a variety of causes, external and internal. To ignore such fluctuations would usually be very costly but to cope with them required either holding

capacities such as stocks, or a margin in the rate capacity or a balance of the two. The most economic balance between rate and holding capacities depended very much on the characteristics of the fluctuation as well as, of course, on the actual cost of having or using the capacities concerned.

Improved quantitative assessments of capacities in relation to their economics, the Chairman concluded, would help in making decisions between alternative ways of meeting a given problem.

The Capacity of Roads

by

J. G. WARDROP*

OF THE THREE types of capacity outlined by the Chairman, road research is chiefly concerned with holding capacities (e.g., car parks, and the maximum lengths of queues at traffic lights before the previous light becomes blocked) and rate capacities. Study of elastic capacity (e.g., traffic generated by road improvements) will certainly have to be undertaken in the future.

THE MEANING OF CAPACITY

Consider what happens when the flow of traffic (the number of vehicles per hour passing a given point) entering a road is increased from zero. Opportunities for overtaking will probably be ample at low flows, so that the average speed will be unaffected. As the flow increases a point will be reached when some overtakings are delayed, and the average speed falls. Capacity can therefore be considered in terms of the relation between speed, or journey time, and flow. It is more convenient to use speeds on lengths of road, and time (i.e., delay) at intersections. But in all calculations of cost the results are treated ultimately in terms of time.

Speeds can be intolerably slow; for instance, in the section of Oxford Street between Oxford Circus and St. Giles Circus, the average speed during the day (not in peak hours) is only $5\frac{1}{2}$ m.p.h. It seems reasonable to say that Oxford Street is loaded beyond its capacity, in any practical sense of the word. If we can accept a particular speed as a minimum, and we know the relation between speed and flow, we can find the maximum flow that can be allowed. Thus, we define the capacity of a road as the flow of traffic which moves at the minimum acceptable average speed.

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* Road Research Laboratory, Department of Scientific and Industrial Research.

VOL.
5
1954

The arbitrary element in this definition is that there is no obvious way of deciding what the minimum acceptable speed should be. It is in fact the question of costs versus benefits which must decide whether or not a road is doing its job adequately.

The Measurement of Capacity

Town Streets. To measure the capacities of a number of streets of various widths in Central London, a test car was driven in the traffic stream at about the speed of the traffic around it. One observer timed its journey with a stopwatch and another counted the traffic that was met. Over eighty runs were made along the roads being studied, at different times of day, equally divided between the two directions.

The streets studied were grouped according to their widths, and the running speed (excluding delay at intersections) was plotted against the flow (Fig. 1). In almost all cases, the speed is unaffected by flow up to a certain point, beyond which the speed falls as the flow increases. The relation can be regarded as a linear one, hence the convenience of using speed rather than journey time.

A simple empirical formula can be fitted to these results:

$$q = 3(31-v)\,(w-6)-430,$$

where q = flow in both directions, vehicles per hour; v = speed, m.p.h. (which must be less than 24 m.p.h.); w = width, ft. This shows that capacity increases linearly with width although not in direct proportion to it. A wide road has a higher capacity than two narrow ones of the same total width. It should be mentioned that these results apply only to two-way streets.

The capacity formula quoted is an average one, and the capacities of individual roads may differ considerably from the expected value. There are many other factors, in addition to the road width, which affect the capacity, such as parking, the flow of pedestrians, particularly on pedestrian crossings, and the presence of large or slow-moving vehicles. The effects of all these factors on certain streets have been studied. As a first approximation, it can be said that they all affect speed linearly. The maximum observed flow of pedestrians on crossings on one 30-ft. street caused a reduction in speed of $3\frac{1}{2}$ m.p.h., while vehicles parked at the kerb caused a reduction of 5 m.p.h.

All the results mentioned were obtained empirically. Theoretical methods would be very useful here, but so far the theoretical approach has not been very successful.

One conclusion which can be drawn from this work is that the capacity of a street between intersections is not a single quantity.

VOL
5
195

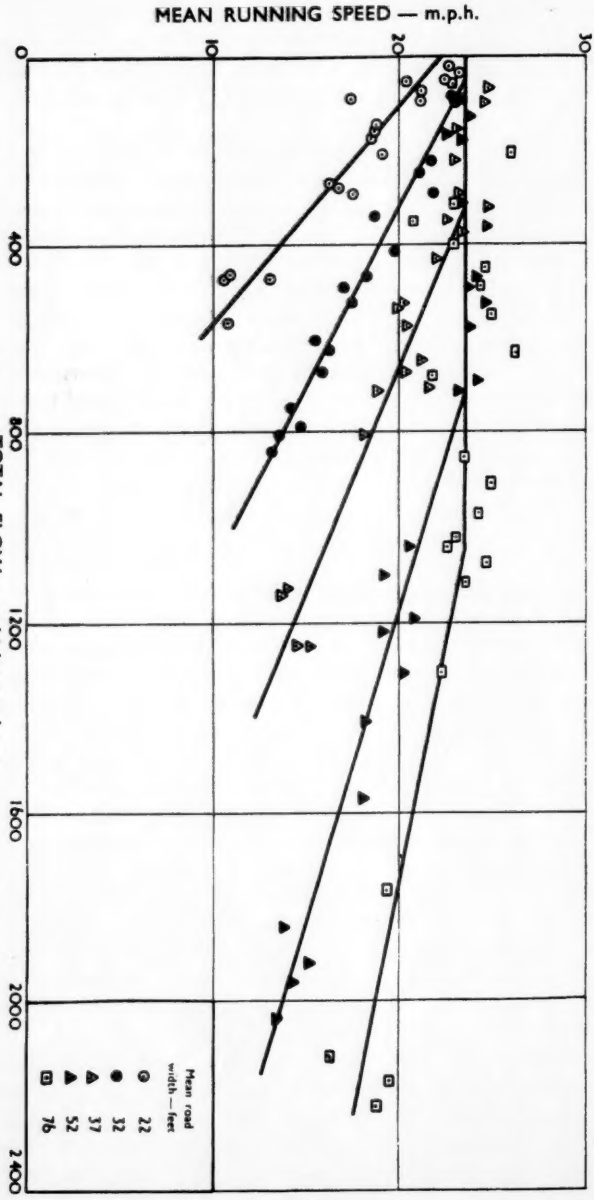


Fig. 1.—Relation between mean running speeds and total flow in Central London

VOL.
5
1954

For planning purposes it is necessary to know the relation between speed and flow over a range of values.

Rural Roads. A somewhat similar approach to rural roads, demands a different experimental technique. On these roads it was easier to measure speeds and flows at a particular point, and this was done automatically. Within the ranges of flow so far observed, speed is almost unaffected by flow. This means that the observed flow is generally less than the critical flow. It is probable that further observations at selected sites will show the expected effect. Speed is, of course, substantially affected by other factors, such as the presence of bends and hills. It is clearly very important to discover the factors that do affect speed, and work on this subject continues.

Controlled Intersections. Controlled intersections in towns are of the utmost importance in determining capacity. On main routes in Central London about 35 per cent of travelling time is consumed in delays caused by intersections. This amounts to about 450,000 man-hours annually per mile during working hours.

Many of these intersections are controlled by traffic signals, of which there are about 3,500 in the country, 900 being in Greater London. Five hundred of the 3,500 work on a fixed cycle of red and green time. The remainder are vehicle-actuated, the timings being influenced by the traffic. Many of the observations on signals apply equally to police control.

The chief difficulty in dealing with signals is the large variation in delay even when the traffic is substantially constant. This is due to short-term fluctuations in the traffic input and the rate of discharge. Because of this variation it was desirable to divide the results obtained at signals into two groups, according to road width. The results are plotted in Fig. 2, which shows that the delay increases very rapidly with increasing flow, and that there appears to be a definite limit to the flow. The limit is reached when the average number of arrivals per signal cycle is equal to the maximum number discharged while the light is green. However, with a signal running in this condition quite small random fluctuations in traffic produce large increases in delay, and a sustained increase of traffic by 5 per cent will cause the waiting queue to lengthen at the average rate of about 25 vehicles per hour per lane, or about 500 ft. per hour (one queue at Staines on a Saturday morning in July 1950 was nearly 1½ miles long and contained about 500 vehicles). Thus, although we can specify a capacity for a signal which is an absolute limit (the "maximum capacity") it is not a practical figure. A heavily trafficked signal works most efficiently when the flow is 80 to 90 per cent of this absolute value.

VOL
5
195

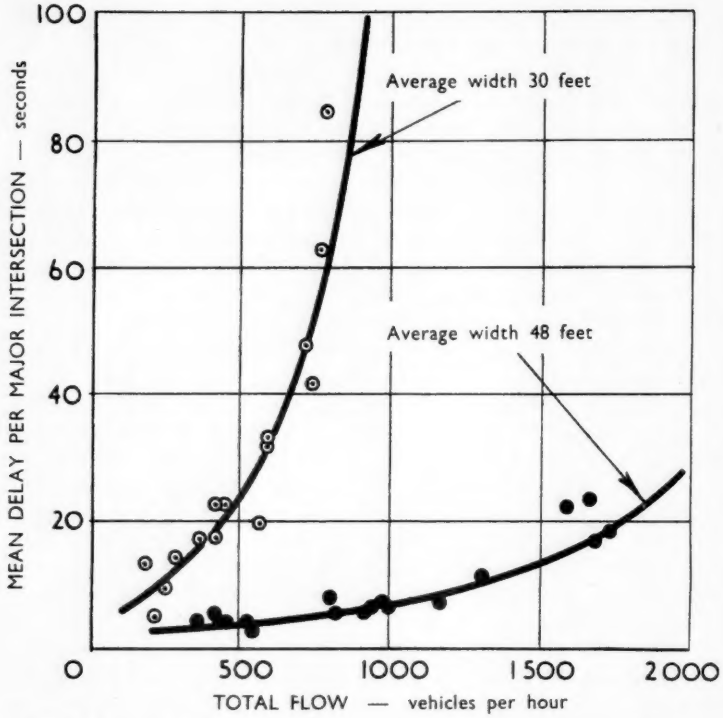


Fig. 2.—Delay per major intersection on two groups of streets in Central London.

Because of the large variations, empirical methods are not very satisfactory. The fact that vehicles are so confined in their movements at signals means that synthetic methods can be applied, using a mathematical model for the behaviour of the traffic, either theoretically or by simulation. So far simulation has been used, both to smooth out observations and to provide predictions. Consider first a fixed-time signal with random input (a random input often being a sufficiently good approximation to actual traffic). We can deal with each approach to the signal in turn, so the problem resolves itself into one with only four variables, the stop time, the go time, the inflow, and the maximum discharge rate. By introducing an arbitrary unit of time we can reduce the variables to three. A computer was built from relays and rotary switches; traffic obtained from tables of random numbers was punched on a teleprinter tape,

VOL.
5
1954

which was fed into the machine at a uniform rate. Each arrival was added to a queue-counter, the signal changes occurred at the appropriate times and, while the signal was green, the vehicles were subtracted from the queue-counter at the appropriate discharge rate. Total delay was found by accumulating the number waiting in the queue at fixed intervals. Comparison of calculated delay values with those observed at different flow levels showed a good agreement. With this machine delay at fixed-time signals can be calculated for any given settings and conditions of traffic. It is capable of working at about three times life-speed.

With vehicle-actuated signals each arm cannot be treated separately, and a simple cross-roads will generally involve at least 12 variables. In order to cope with the control system, the pilot model of the Automatic Computing Engine (ACE) at the National Physical Laboratory has been used. It can work at an effective speed of six times life-speed on a complete four-way vehicle-actuated signal.

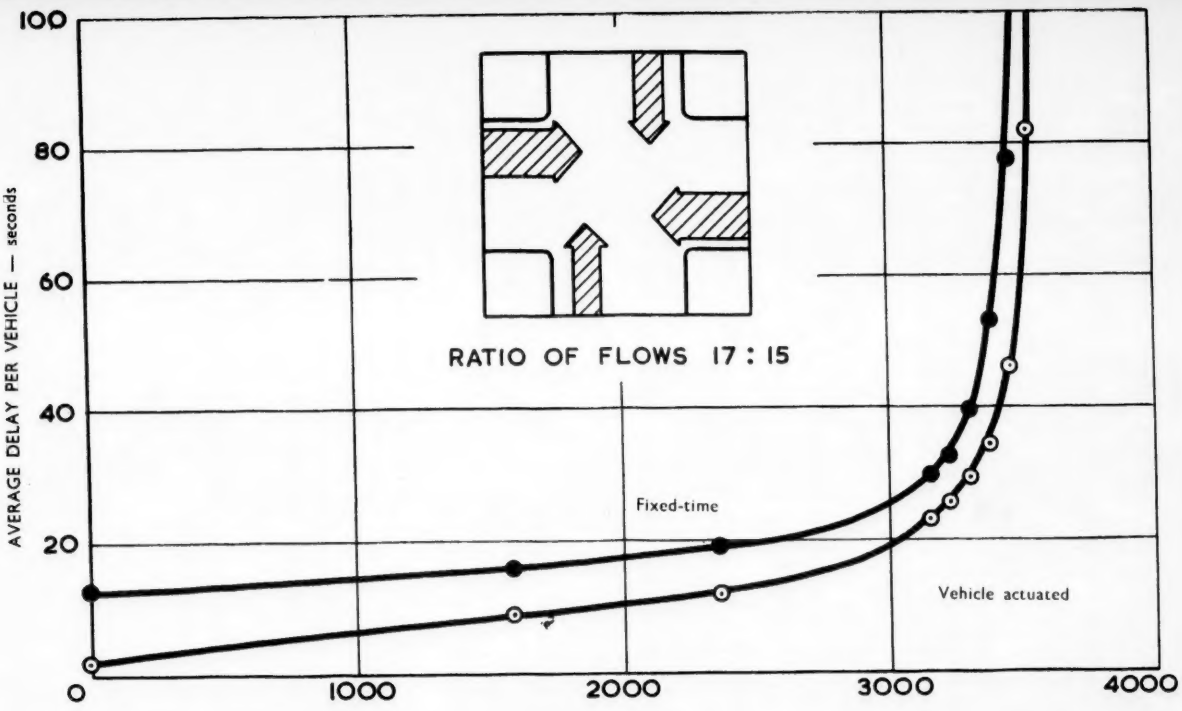
Other studies have been concerned with the effects on the capacity of signals of road width, right-turning vehicles, vehicles parked near the stop line, and the proportion of medium and heavy goods vehicles. It has been found, for instance, that capacity is roughly proportional to road width.

The Uses of Capacity

In planning new roads and road improvements, the amount of traffic which will use them is usually to some extent predictable. It is desirable to know what widths of roads and types of intersections are required. For a given road width and a given number of intersections per mile, we can build up the journey speed/flow curve for the street as a whole from the results obtained in Central London, using the running speed/flow and delay/flow curves in the appropriate proportions. From the resulting curve we find the capacity for a given speed.

An example of the kind of result we get by using computing methods to simulate traffic signals is shown in Fig. 3. This compares the delay/flow curves for a fixed-time and a vehicle-actuated signal, using identical traffic. The vehicle-actuated signal shows a marked advantage, but the difference expressed as a proportion of the delay is less as the flow increases, and the two curves approach one another asymptotically. This shows that in some cases the vehicle-actuated signal behaves much like a fixed-time one when it is heavily loaded, and has the same ultimate or "maximum" capacity. We have found, however, that if the flow pattern changes from that for which

VOL
5
195



Fixed-Time	Vehicle-Actuated (S.G.E.)
Green Each Phase 40 Seconds	Max. Green Each Phase 40 Seconds
Cycle 86 Seconds	Vehicle Interval 4 Seconds

Fig. 3.—Comparison of Fixed-time with vehicle-actuated signals

VOL.
5
1954

the signals are set, the vehicle-actuated signal is able to adjust itself in such a way that its maximum capacity is greater than that of the fixed-time signal.

We have used this computer technique to determine the settings giving the least overall delay for fixed-time signals, and we hope soon to publish the results, including tables of delay under all conditions that are likely to be met.

The question of balancing capacities is important on roads, and since the intersections are usually the critical points, it is these which should be matched along a road carrying a constant flow. A good example of balance between holding capacity and rate capacity is a traffic signal which has another signal nearby. The road widths may be such that the signal would work quite satisfactorily in isolation but the queue would sometimes run back into the neighbouring intersection and cause trouble there. If so, it might be necessary to increase the width of the appropriate arm of the intersection, thereby increasing the rate of discharge and shortening the queue.

To avoid the arbitrary factor of "minimum acceptable speed", and obtain a firm basis for decisions, capital investment and profit must be taken into account, balancing the cost of improved or new roads against the financial benefits to be expected from savings in time and materials.

As an example two specific instances of proposed improvements in London may be considered. The first is the widening of a 950-ft. length of road from an average width of 36½ ft. to a uniform 45 ft. This road carries a flow of 1,600 v.p.h. at a speed of 14 m.p.h. Using the formula relating speed, flow, and road width, we should expect the widening to result in an increase in speed of the order of 4½ m.p.h. We can now calculate the expected annual savings in terms of man-hours, the time of goods vehicles and buses, and also of fuel, using an estimate of the relationship between fuel used and time saved. The resulting savings and the cost of the improvement are given in table I. Two extreme values for a man-hour, 1s. and 5s. are given in the table, the true value probably lying somewhere between the two. The rate of interest on the capital expenditure will therefore lie between 1½ and 4 per cent, and it may be concluded that the improvement is not justified as an investment. If in the future the traffic increases by 20 per cent, the rate of return will be between 3 and 9½ per cent. Even so the case for improvement can hardly be considered outstanding as far as savings in time and fuel are concerned. Moreover, it is extremely unlikely that traffic on this street will increase by 20 per cent for many years; it would only be possible if the intersections feeding the street were improved as well.

VOL
5
195

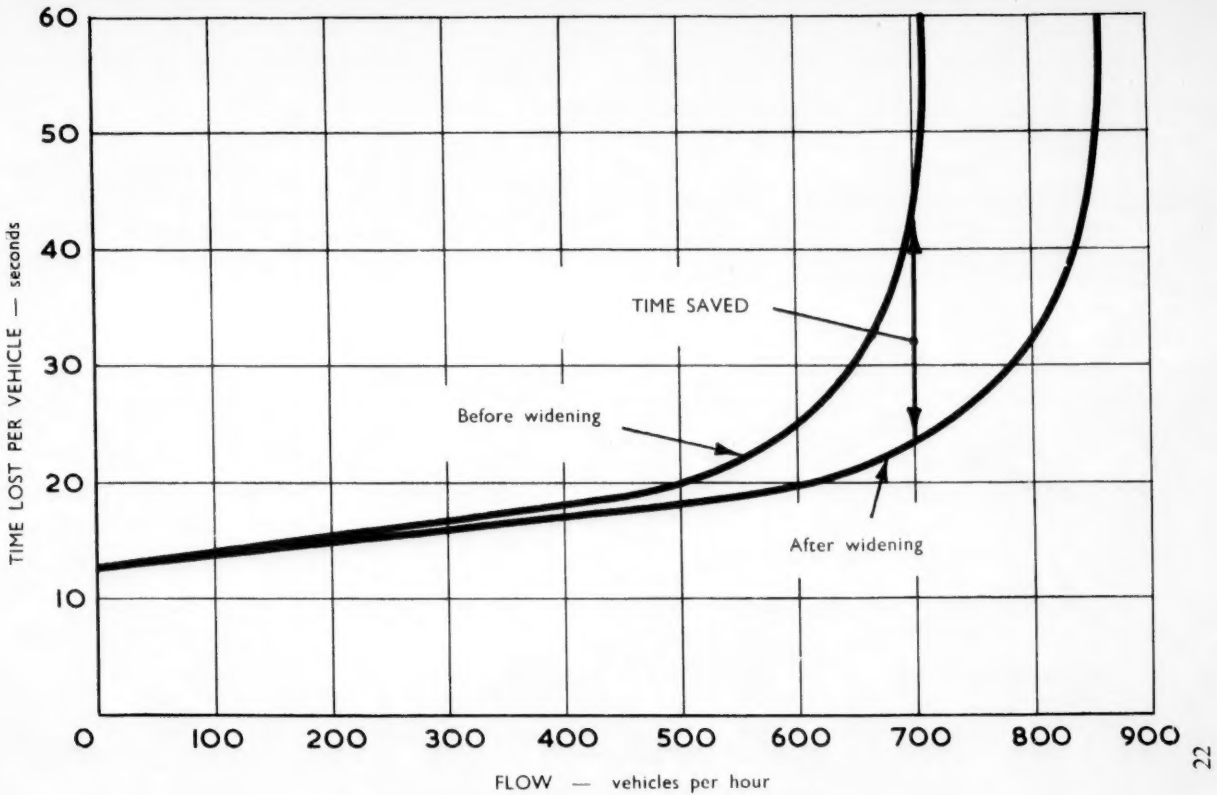


Fig. 4.—**Effect of widening an intersection by 20 per cent.**

VOL.
5
1954

TABLE I

The value of widening a road from 36½ ft. to 45 ft.
(Capital cost: £460,000)

		Expected annual savings assuming man-hour worth:	
		1s.	5s.
Man-hours (64,000)		£3,200	£16,000
Time of buses		£1,800	£1,800
Time of goods vehicles		£700	£700
Fuel		£500	£500
Total		**£6,200**	**£19,000**
Rate of interest (per cent)	Present traffic	1½	4
	20 per cent increase in traffic	3	9½

As a contrast, the second improvement is the small widening, by 3 ft., of a 14½-ft. approach to a traffic signal. In this case two delay/flow curves were drawn (Fig. 4) using the delay computer, and the savings were calculated, allowing for the traffic variation throughout the day. The resulting savings are shown in table II. Here the value of the man-hours saved is omitted altogether because the savings in the time of buses and of goods vehicles and in fuel alone represent a rate of interest of 18 per cent with present traffic. There is therefore little doubt that the improvement is worthwhile.

These examples have shown how road-capacity measurements can be used, and their importance in planning an economically sound

TABLE II

The value of widening one approach to an intersection from 14½ ft. to 17½ ft.
(Capital cost: £5,080)

	Expected annual savings
Man-hours (11,600)	—
Time of buses	£640
Time of goods vehicles	£160
Fuel	£130
Total (excluding man-hours)	**£930**
Rate of interest with present traffic (per cent)	18

road system. There are many more problems to be tackled, and more data are needed to extend the existing information.

ACKNOWLEDGEMENTS

The investigations described form part of the programme of the Road Research Board, and the paper is published by permission of the Director of Road Research.

The author is indebted to the National Physical Laboratory and to his colleagues at the Road Research Laboratory for their part in the investigations.

Capacities in a Manufacturing Process

by

R. SOLT*

STEELMAKING is taken as an example of a manufacturing process in which capacities play an important part. A works is considered with a melting shop in which ten 100-ton furnaces are at work converting scrap and cold pig iron into steel. The weekly output should be about 15,000 tons, that is equivalent to an average rate of about 90 tons per hour.

In the process as a whole, the raw materials are loaded at one end, they are converted to steel in the furnaces, and cast into ingots, which are rolled down in the mill at the other. At one end, loading can be arranged to go on at whatever rate we please; at the other the rolling mill deals with ingots at a fairly even rate. Unfortunately the middle stage of the process, open-hearth furnaces, produces steel a batch at a time, and it is difficult to keep the time taken by successive charges constant. In consequence, furnaces often "bunch". If all the furnaces are ready to accept charge at about the same time, then some 1,000 tons of scrap and iron must be put into the furnaces in the space of 3 to 4 hours, and the handling capacity must accommodate a flow of 250 to 300 tons per hour.

A few hours later, 1,000 tons of finished steel will flow out of these furnaces and will be cast into ingots for the mill. This weight of steel, produced in a very short space of time, will be processed by the rolling mills at the average rate of 90 tons per hour, over the next ten hours or so. Meanwhile, such ingots as are not immediately wanted must be stored somewhere to retain their heat for rolling.

The handling capacity from the furnaces into storage must be sufficient to deal with the maximum flow of ingots; handling capacity

* British Iron and Steel Research Association.

VOL.
5
1954

out of storage to the mill need only be sufficient for the average flow rate. The storage capacity must be sufficiently great to store the excess of what is made over the demand.

This is a typical case of a batch process which feeds a continuous process. It can be represented by a gate with variable opening (or a gate at which the passengers—i.e., the steel ingots—arrive in random batches) which feeds a fixed gate through a temporary storage.

The 1,000 tons of material which make up a complete charge for all the furnaces must previously have been loaded into boxes. We can ourselves determine the rate at which these boxes shall be loaded. Our choice will lie between two extreme conditions:

(i) Loading can be done at a steady average rate of about 90 tons per hour, and any material not wanted at the time of loading is put into temporary storage; the storage capacity must in that case be sufficient to take about 600 tons of material.

(ii) Loading can be done at the rate at which the material is wanted; so that loading proceeds at a very high rate (250 to 300 tons per hour) for 3 to 4 hours and then not at all for the next 6 to 8 hours. Very little storage capacity is needed, but a large amount of loading equipment will be necessary, much of which will be idle most of the time.

The most economic solution will almost certainly lie somewhere between the two extremes. It is reached when the marginal cost of more storage capacity is balanced by the marginal cost of a higher working rate (i.e., of higher rate capacity). As we shift the balance from one extreme to the other, we presently arrive at a condition when it would cost just as much to raise the storage capacity as the working capacity. As much as possible of each demand peak is then met by working to capacity, and the remainder is satisfied from stock, which is replenished during the subsequent slack period.

It may well be uneconomical, however, to cater for peak demands. In practice, it is most unlikely that all furnaces should be wanting to do the same things at exactly the same time: varying degrees of bunching are liable to arise with varying degrees of probability.

We must therefore discover the probability with which varying degrees of bunching are likely to occur. Severe bunching cannot continue indefinitely. Each furnace must complete one cycle before it can start on the next; if the cycle time is 10 hours, we know that all the furnaces in the shop can go through any stage of the process only once in any 10-hour period. When a busy spell occurs it must therefore alternate with a slack spell within the 10-hour period.

When a bunching condition arises which the system cannot cater

for, i.e., when the demand becomes too great for us to meet, then we lose some production. What is needed is an economic balance between the cost of the system, and its value as a service to the process. This economic optimum is reached when the marginal cost of making the system more efficient is equal to the marginal return which this increased efficiency brings with it. It is a problem of finding a balance between storage and rate capacity on the one hand, and of matching capacity as a whole to the probability of demand on the other.

There are many parallels between the system described and systems in other fields. The similarity of the basic problems that they present is, in the writer's opinion, worth stressing.

The Flow of Motor Spirit from Refinery to Consumer

by

S. SHRAGAH*

AS A RESULT of great expansion in the United Kingdom's refinery capacity since 1947 the whole of this country's requirements of motor spirit are manufactured in this country. This development has necessitated reorientation of the distribution pattern, which is nevertheless based on existing installations and depots. This network has grown up since the earliest days of oil distribution. It comprises, besides the refineries themselves, "ocean installations" such as those on the Thames, Mersey, Clyde and Humber, and at Avonmouth, and smaller inland storage and distribution centres.

The sizes and speeds of delivery vehicles are components of the distribution system's capacity. From 1941 to October 1953 the maximum permissible capacity of a road tanker for motor spirit was 3,600 gallons, it is now 4,000 gallons. The industry is standardizing its fleet concentrating on the largest vehicles for "bridging" to inland depots and utilizing smaller vehicles of 2,000 gallons for re-delivery to the trade. The speed limit for both these classes of vehicles is 20 m.p.h.

Another important factor is the tendency for garages to take all their supplies from one company or group of companies. This reduces the number of stations to be supplied by one company and increases the load that can be delivered to the individual stations.

* Shell Mex and B.P. Ltd.

VOL.
5
1954

The general principle governing the holding of stocks must be the holding of the minimum amount commensurate with the operations, marketing conditions and sense of public responsibility in the industry. Underlying these factors and inter-related to them is also the economic aspect which involves calculation of the cost of providing storage and transportation equipment of varying sizes in relation to the cost of distribution obtained.

Assuming ideal conditions the formula governing storage capacity will need to embrace the following factors:

(1) Rate of daily issues. This would allow for issues at peak periods and for anticipated trade changes.
(2) Size of the economic transportation unit used for moving product (the E.T.U.).
(3) The frequency with which replenishments must arrive to keep pace with issues. This is a function of E.T.U. and daily issues and can be expressed in an equation:

$$\frac{\text{E.T.U.}}{\text{daily Issues}} = \text{frequency factor of replenishment in days.}$$

(4) Insurance cover against breakdown or delays, e.g., by weather.
(5) Trading value placed on possessing further reserves to ensure maintenance of business against *force majeure* in the widest sense, e.g., labour disputes.

Commercial Consumers' and Dealer Sites. The oil companies aim is to base their service on a minimum of 24 hours' notice, and do not usually deliver on Sundays. The assessment of daily issues, therefore, ought to be based on peak sales and the storage capacity should allow for the absence of replenishment on Sundays and for the 24-hour minimum service.

The E.T.U. will be the holding capacity of the delivery vehicle and it is assumed that arrangements have been made to order in quantities corresponding to the capacity of the E.T.U. Thus the formula for optimum storage becomes:

$$\text{E.T.U.} + 2 \text{ (peak daily issues)}$$

to which should be added insurance cover for breakdowns, or delays to replenishments and for possession of additional reserve. The amount of storage for these last two items will largely be a matter of individual assessment and in practice might well result in the addition of storage equivalent to a further two days' peak issues.

Road-fed Depot. The daily issues from a depot will tend to remain on a steady level. At a road-fed depot a 24-hour minimum service will apply just as for dealer sites and allowing for the absence of replenishments on Sundays the formula for depot storage can be expressed as E.T.U. plus twice peak daily issues. The usual insurance cover has to be applied which may well result in the storage capacity of the depot being E.T.U. plus 7 days' peak issues in recognition of the greater responsibilities borne by the depot.

Rail-fed Depots. Here the E.T.U. is the railcar which usually holds approximately 4,000 gallons of motor spirit. The speed of replenishment is more difficult to determine because although daily orders corresponding to sales are lodged on and despatched from the supply point, experience shows that the relationship between despatch times and arrival times is not a constant factor as the traffic tends to bunch. The optimum storage must therefore allow for the bunching factor and the formula for optimum storage will become E.T.U. plus bunching factor in days multiplied by the daily issues. On top of this the usual allowance has to be made for insurance cover which may well result in the storage capacity of this type of depot being E.T.U. plus 10 days issues.

Coaster-fed Depot. Here the speed of replenishment is more difficult to calculate as vessels cannot be expected to operate with the same degree of precision as road vehicles. The hazards and delays caused by wind and weather greatly increase the allowance to be made for insurance cover against interruption to replenishment and often this factor can vary at different points.

Sales at a coaster-fed point will include the sales of any dependent depots and the formula for optimum storage will be E.T.U. plus daily issues multiplied by the time factor for replenishment plus insurance cover.

In the case of shipping movements the E.T.U. is determined by balancing considerations of water conditions, port facilities, volume of trade and the cost of appropriate storage against the freight rates of various sized vessels capable of supplying the reception points.

Specialized types or sizes of vessels are sometimes necessary to serve ports which are subject to physical restrictions or particular regulations but where these conditions do not apply it has been established that the overall cost considerations and the advantages to be gained in flexibility of operation justify the employment of a fleet consisting of a limited number of types rather than a large number of types each designed solely for one or two reception points.

VOL.
5
1954

Ocean Installation. The ocean installation can be treated in the same way as a coaster-fed depot though in view of its greater responsibilities, a wider margin if insurance cover is desirable.

Refinery. Lastly the refinery itself. The daily issues will be the aggregate of demands for direct delivery, transportation to road/rail fed depots, coaster-fed points and installations. Its speed of replenishment will be the daily rate of production. As refineries manufacture a whole range of products from crude, the rate of production for any given product is liable to fluctuate at different times according to fluctuations in the demand for other allied products. Allowance must be made for this variable in calculating storage capacity. Although the refinery is a producing point and not a receiving point for motor spirit, the optimum storage still has to take into account provision for the E.T.U., although here the principle works in reverse. The storage must be adequate to cater for the maximum quantity the refinery is likely to be called upon to ship away at any given time to its dependent installations. This is governed by a close calculation of the replenishment cycle in conjunction with the E.T.Us. for its implementation.

The insurance factors at a refinery must take into account that continuous running of the refinery plant is vital to production costs, remembering the high level of capital investments in these complex units which may well have a relatively limited working life in an industry faced with constant changes and improvements in technique. Protection in the shape of added storage capacity must also be taken against the possibility of interruption or breakdown to the refinery plant. The amount of this depends upon the industry's sense of public responsibility and the importance attached to the maintenance of supplies for running its business.

In considering the establishment of secondary distribution centres the industry takes advantage of the long coastline which we possess and the fact that use can be made of certain inland waterways. The cost of moving motor spirit by water in large consignments is considerably cheaper than the cost by road or rail.

In certain inland areas the industry uses secondary centres of distribution which are supplied by road or rail. Usually topography, e.g., the mountainous parts of Wales or the Lake District or the incidence of dense pockets of trade, e.g., at Sheffield, governs the decision to operate from such depots, although quite often they can be justified on economic grounds where favourable rail rates from the refineries can be obtained.

The proportion of working capital tied up in stocks of motor spirit must vary considerably between the different companies. It is

particularly difficult to give any figure which might be regarded as a representative average because distribution systems vary and because of the differing extent to which products other than motor spirit are handled, since in the latter case storage and equipment in many respects are common to more than one product. The figure in the case of motor spirit, however, is kept down to a minimum by making use of bonded storages wherever possible. To qualify for the privilege of bonding, depots have to possess a minimum storage of 200,000 gallons for dutiable products. The present-day concentration of distribution from fewer and larger centres makes it possible to utilize bonded premises to a much greater degree than formerly. The benefits of this are reflected not only in the reduction of capital tied up in stocks, but also in the fact that working losses and leakage losses are in respect of the value of the product only without the addition of the 2s. 6d. per gallon duty. The closest attention is paid to operational procedure and the types of storage used at depots in order to minimize leakage and handling losses, since in aggregate the yearly losses from such causes can represent a formidable addition to distribution cost.

Discussion

Mr. TODD (Imperial Tobacco Co.), dealt with the choice between increasing rate capacity to allow for seasonal fluctuations in demand for cigarettes, and establishing enough holding capacity to allow an even production rate throughout the year. The additional cost incurred in producing a given quantity of cigarettes by working overtime was approximately equal to the cost involved in carrying the same quantity of goods in the form of duty-paid stock already manufactured for between three to four weeks. When worked out theoretically, fluctuations in the rate capacity were so small that they were not worth while because of complications in other directions. In short, those things could not be done to the theoretical optimum; it was only worth while departing from a simple system if something positive was obtained. The cigarette trade did not get enough from putting a small contribution in the rate capacity, so they took the whole thing out in a holding capacity.

Mr. H. G. JONES (Steel Co. of Wales Ltd.) quoted the experience of a steel works faced with the decision of buying oxygen at 5s. per 1,000 c. ft. with a high minimum consumption and no variation, or at 7s. 6d. with all the required fluctuations taken into account. The problem was to determine the economic base load and balance the two supplies. A frequency diagram had been constructed based on quarter-hours and an equation fitted to that curve enabled the

VOL.
5
1954

optimum level to be evaluated. It had been predicted that it would be cheaper in the long-run to allow a fairly high level for the 5s. per cubic ft. of oxygen even if it had to be blown off to atmosphere. He gave that as one example of a balance between the base load and usage.

The CHAIRMAN thought that a good example, because normally to blow off a valuable commodity such as oxygen would fill managers with horror and they would go far to avoid doing so rather than balancing the two. Very often it was an advantage to waste something at one stage if a much larger saving could be accumulated over a long period.

In discussion on *The Capacity of Roads* several speakers suggested that Mr. Wardrop had left an important factor out of his equations by not sufficiently considering safety. Various suggestions were made by which road carrying capacity might be effectively increased, one interesting report coming from Chicago where hydraulic curbs are raised or lowered at peak periods to provide more capacity for traffic in the prevailing direction. The segregation of traffic of different speeds into different lanes was also suggested. Both Mr. WARDROP and the CHAIRMAN suggested that an improvement in road manners was the shortest cut to this desirable result.

Mr. WARDROP said that had confined his paper to the consideration of capacity in relation to delay, but it was quite wrong to suppose that no regard was being paid to the importance of reducing the accident rate. Few people would be willing to improve roads to increase the speed of travel if convinced that this would also raise the accident rate. The Laboratory was investigating the relation between death and injury rates and a number of road characteristics.

The CHAIRMAN commented that the hazard of accidents and the problem of delay were factors dominating the discussion of capacity; throughout the country their costs were of the same order of magnitude and they were of equal importance, on any cost basis, of a reasonable assessment of the amount of money or man-hours worth spending to save human life. The speed at which people drove was dependent on their appreciation of the situation confronting them; they mentally balanced their economic urgency to get on against the hazard to themselves and (it was hoped) to others of driving faster. Accidents were caused largely by the tail of a distribution curve of those who did not succeed in appreciating situations early and often enough. Ninety-nine per cent of the decisions taken in this country in respect of roads were taken because of an accident at a particular spot; gross maladjustment at

traffic lights would probably be due to a decision taken after a fatal accident there years before and persisted in despite all scientific evidence.

In discussion on *Capacities in a Manufacturing Process* consideration of "bunching" led to comments on Ashcroft's tables concerned with the problem of how much material would flow out of one kind of machine if subject to a certain rate of breakdown, the breakdowns themselves being produced by chance and occurring irregularly. The build-up of material between such machines could be dealt with if the distribution of the breakdown times of the machines were known. This could be found by going to Ashcroft's original formula. His tables gave the average of the breakdown times, but what was wanted was the distribution of the breakdown times from which could be worked out the minimum storage to keep those machines going and the maximum speed at which to run to prevent piling up.

It was suggested that the pattern of flow from steel furnaces causing the need for the buffer stock could be subjected to mathematical treatment by a formula, depending on the size and cycle of the furnaces, which would work out in practice within certain statistical limits. If mathematical expression could be given to it; would it be an economic proposition to try to use the statistical limits within which that formula was oscillating to control the way the furnace would operate? Would it be economical to retard a furnace in order that it should not cause trouble at that buffer? Or, if the process were random, could it be treated by statistical methods to give a rigorous answer to the question of the size of the buffer stock?

Another speaker thought there should be a balance between variations in rates of production and one single holding capacity. In the nylon industry, for example, yarn was held against customer demand. Two separate holding methods were wanted, one for day-to-day use, the other providing for the storing of long-term emergency reserves which, while expensive for day-to-day operations, was cheap for an annual operation.

Mr. SOLT in reply said that definition of the probability of the various gaps between furnace tappings occurring one after the other was precisely what had been attempted. A formula had been produced and used, and comparison with actual cases showed it produced very close results. It was in fact based on an assumption of randomness in the arrival of the furnaces. Although he was modestly hesitant about releasing the formula to statisticians, to everyone's surprise it had given results that actually occurred in

VOL.
5
1954

practice and it was being used. It was not too simple, because having the interval between succeeding furnaces was not sufficient as that did not give the full picture; it was necessary to find the intervals between the first and the last of groups of furnaces, with the groups varying in size up to the number of active furnaces in the shop. This should give a complete picture of bunching. It was more difficult to reconcile that result with what could be done in practice. Attempts had been made to schedule the furnaces, but he knew of only one successful attempt. Once bunching occurred, the question was to find out how much delay, and how often the condition occurred.

In his reply to the discussion Mr. SHRAGAH said that it was very hard to divorce reserve stock in a petrol storage from normal operating stock without having two physically divorced premises in any one large storage centre. Though the tanks not used for day-to-day purposes might be isolated, there came a time when it would be necessary to turn over the stock in what might be called non-operating or reserve tanks. The period of time would be a varying one. At one period petrol stocks in any non-operating stage were "turned over" at least once a year, but he was not able to say what was the frequency with the modern additive type of petrol.

Assessment of storage was, he thought, largely on the basis of common sense and practice. It related, although not highly statistically, to the types and capacities of replenishment units, and their speed and method of operation, in relation to the volume of trade from any given storage.

ABSTRACTS AND REVIEWS

The Frequency and Importance of Errors in Invoices Received.
ROBERT H. GREGORY.
Accounting Research, 1952; **3**, 332.

"This article deals primarily with the question: Is it necessary or even desirable to process in the same way all invoices received from vendors? The broader question at issue is whether the accounting and auditing techniques that are desirable under certain conditions are, in fact, applied beyond the point of limiting returns". (Opening paragraph of the original.)

The author drew a random sample from the invoices received and processed by an automobile factory in a seven months period, and analysed these according to date, name of vendor, amount of invoice, type of error, and amount and direction of error. (A comparable analysis of amount of invoice only for 1950 suggested that

the size distribution was fairly stable.) The percentage of invoices containing errors appeared to be roughly independent of the size of invoice in each of the three main sorts of error, and to be independent of the vendor. Although the vast majority of errors were in the vendor's favour, the cost of identifying and correcting them greatly outweighed the loss to the company except in the case of the very large invoices, which form only a small proportion of the total. It is suggested that the most economical procedure would be to check all large invoices, but only a sample of smaller ones to check that the number and size of errors was not on the increase.

Fitting the Job to the Sailor.
K. F. H. MURRELL.
Occupational Psychology, 1 *January*, 1953; **27.**

This paper is a review of the development of the Naval Motion Study Unit and its work over the years 1947–1952. The changes in the jobs tackled over these years reflected the change towards a greater mechanization of operations. In all cases the importance of studying human work in its actual setting is emphasized. Examples given include the analysis of gun drills, aircraft handling on the flight decks, and the washing habits of seamen. Three general ideas emerge from the review. Firstly a unit such as the one under consideration should contain individuals from a number of disciplines who should work together. Secondly such a unit should be located inside the organization it studies, and thirdly close contact should be maintained with university research workers.

Establishing a Suitable Stoker's Premium.
L. H. LE POOLE.
Tydschrift voor Efficientie en Documentatie, 1953; **23**, 19 *and* 82. (*In Dutch.*)

The author explains the difficulties of using the direct measurement of boiler efficiency as a basis for the calculation of a stoker's premium. He then develops (for coal fired boilers) graphs indicating the heat losses in flue gases and ashes as a function of the percentage of CO_2 and $CO+H_2$ in the flue gases and the percentage of combustibles in the ashes. For a typical boiler installation these graphs are then used for the construction of another graph giving the stoker's premium as a function of the percentage of CO_2 and $CO+H_2$ in the flue gases and combustibles in the ashes. This graph takes into account the fact that with increasing boiler efficiency any additional increase is progressively more difficult to realize.

VOL.
5
1954

In a second article the author develops similar graphs for oil-fired boilers.

Mathematical, or "Linear" Programming: A Non-mathematical Exposition.
ROBERT DORFMAN (University of California)
American Economic Review, December, 1953; **43** (5.1), 797–825.

The principles of mathematical programming are illustrated by graphical representation applied to a simple, hypothetical example of an automobile firm. The similarity and differences between this method and the classical economic approach to the problem of deciding the best of two (or more) factors of production are discussed.

A short section gives examples in which the method has been applied in practice.

A Note on Forecasting Demand for Special Offers in a Magazine.
MAX K. ADLER.
J. Industrial Economics, November, 1953; **2** (1), 44–50.

The problem concerns the forecasting of the demand for special offers made by the weekly magazine *Woman*. If the demand is over-estimated some of the articles offered are left on the hands of the publisher, while if it is underestimated all the requests sent in by readers cannot be satisfied.

A method of forecasting is derived based on a sample survey, the survey estimate being adjusted by a factor composed of (a) the ratio of the survey estimate to actual demand for a previous offer of similar class of article and (b) the ratio of the price of the article previously offered to the price of the current article.

Forecast demands are compared with actual sales for eight recent offers. Reasons for differences are discussed.

Operational Research—A New Tool for Management.
I. S. LLOYD (South African Board of Trade).
J. Industrial Economics, July, 1953; **1** (3), 175–186.

This paper reviews O.R. as the author saw it in the course of a survey made in Great Britain in 1951. He discusses what O.R. is, what it aims to do and how it can be organized and fitted into the management structure.

VOL
5
195

Nuevas aplicaciones de la Estadistica: La investigacion operacional: (New Applications of Statistics: Operational Research).
SIXTO RIOS.
Trabajos de Estadistica, 1952; **3** (3), 255–272. (*Spanish.*)

This paper, read at the inauguration of the School of Statistics of Madrid University in October, 1952, gives a general review of the development of O.R. It refers briefly to war-time examples and indicates some industrial applications. Principles and methods are discussed and directions in which it may be useful are indicated.

Fourth Report on the Australian Stevedoring Industry Board.
Sydney, Australia, 1953.

An excellent operational research study is set out in the Appendix to this report, entitled: "Report on the railage and stevedoring organization for the loading of steel at Port Kembla: based on an investigation into the loading of the vessel 'Iron King' between 8th and 24th April 1953". The aim of the research party from the Board's staff, including a port inspector, was both fact-finding and therapeutic. As might be expected, the key factors turned out to be diverse. They ranged from the organizational planning for stowage in the ship's hold to the availability of tractors —or crowbars—for moving rail trucks during loading; to the layout —and disrepair—of rail-tracks, and to the existing customs in regard to the use of locomotives in shunting and sorting trucks.

Moreover, whilst the port order provided for three shifts of stevedores (although the investigation found only two shifts being worked), the locomotives were hired from the Government Railways and were not available for a 24-hour service; and whilst the investigation gave much attention to the supervision of wharf-labour, that of locomotive crews was non-existent. The report records that during this special investigation, delays were less than usual owing to the special efforts made by exporters, stevedores, and those responsible for shunting, though this manifestation of the "Hawthorne effect" was offset by a total of 3 days' unauthorized stoppages thrown in by the local Union branch. The Board's report is able to record that although these matters were not resolved immediately or simply, and a great deal still remains to be done, a real improvement resulted. Thus, the labour force was adjusted to the need and a 24-hour rail service was arranged. Whereas in November 1952 the shipment of steel from Port Kembla was 34,000 tons, in June 1953 it was 65,000 tons. R. G. S.

VOL.
5
1954

O·R

OPERATIONAL RESEARCH QUARTERLY

VOL. 5 NO. 2 JUNE 1954

Contents

Supply and Demand 37
Reflections on Operational Research——E. C. Williams . . . 39
A Survey of Operations Research in the U.S.A.——Ellis Johnson . 43
Abstracts and Reviews 48
Road Research 50
Large Number Research——Manfred Knayer 54
Recent Courses 56

EDITORIAL NOTES

Supply and Demand

The two papers in this issue by Dr. Ellis Johnson and by Mr. E. C. Williams both refer with justifiable anxiety to a danger common to this country and the U.S.A. This is, that the demand for operational research may exceed the supply of experienced competent practitioners. Dr. Ellis Johnson's paper indicates that this may already be the case in the U.S.A. The survey made in this country last year by Goodeve and Ridley (*Operational Research Quarterly*, June, 1953; **4** (2), 21–24) did not indicate that this was then so here; the survey, however, was not specifically designed to determine this point and it is known that a number of developments of operational research have taken place since. The results of such a situation would be unfortunate. In the first place much desirable, and indeed necessary, operational research would remain undone; in the second place the quality of the work done would be diluted by the employment of unsuitable people to undertake it. The second result may in the long run be the more serious. Its effect might well be to give operational research a reputation for charlatanry, and to cause it to suffer the same fate as has befallen some other attempts to bring science to bear on problems of organization and management.

Two general lines of approach to this problem may be discerned. The first is the negative but necessary approach of setting up professional standards to prevent the inexperienced from passing themselves off as competent. The Operational Research Societies both in this country and in the U.S.A. limit their membership to active

operational research workers, and to that extent exercise the only check. The second approach, fully accepted by both Societies as part of their function, is to develop the supply of suitable workers. It is interesting to note that doctorates in the subject have been instituted at the Massachusetts and the Case Institutes of Technology. Operational research, however, is a method of approach to problems rather than a collection of techniques and the scope for direct academic instruction is limited. It appears that the doctorate courses mentioned recognize this by being open to scientists who wish to study practical operational problems rather than concentrate on laboratory or theoretical work.

It may be hoped that this approach, which so far has not been developed in this country, may lead to a useful supply of operational research workers in the future. Immediately, however, there is a need to attract into the field a greater number of established scientists and this can perhaps best be done by demonstrating to them that it provides opportunities for work which is worthy of their ability and training. This may lead to something of a vicious circle, however, since managements who are considering making use of operational research may hold back until they are confident that suitable workers are available to undertake it and scientists who are secure in other fields will not come forward until they are satisfied that there will be work for them to do. To this end short courses such as that organized by the Institute for Engineering Production at Birmingham University—and to a less degree other conferences and meetings—provide a useful service. By bringing together samples of good work that has been done and by illustrating some of the techniques that are employed such courses indicate the satisfaction that can be derived from doing the work and at the same time can kindle interest in operational research among potential users and can set up in the minds of those attending, both practitioners and users, comparative standards by which the quality of future work can be judged.

There is no short answer to this problem. There need be no doubt, however, that the necessary workers will be forthcoming in the long run by the extension of formal training and above all by the continued application of operational research in established organizations and by the movement of workers, as they become experienced, from established organizations to new ones.

Operational Research in France

We note with interest the establishment of the Centre Française de Recherche Operationelle, at 11 Rue de Longchamps, Paris 16e. We wish it every success.

VOL.
5
1954

REFLECTIONS ON OPERATIONAL RESEARCH

by

E. C. WILLIAMS*

THE SUBJECT of this talk was given as *Education and Training for Operational Research*. It was soon obvious to me that I could not discuss that subject without making clear what it was that I thought we were educating and training for. I was thus forced, much against my will and better judgment, to enter the ranks of those who have tried to say what is the nature of operational research.

The whole of my own experience is in military operational research. I have no first-hand experience of industry or of the nationalized corporations. Whatever I say may have no validity outside military operational research, and only those with experience in industry will be able properly to judge whether my views can be extended to cover their activities.

The term operational research was specifically coined, by A. P. Rowe, to describe the activities of a small section of the Air Ministry Research Station at Bawdsey in the years 1937–39. The word "operational" was put into the title to distinguish the activities of this particular section from the normal research and development work of the Establishment on the various forms of radar; the word denoted, correctly, that the work to which it was applied was concerned with the using of the radars, in the widest sense, as a part of the air defence system as a whole. Earlier work on the methods of fighter interception carried out independently by B. G. Dickins was, equally well, operational research although it was not, at the time, called by that name. After the Bawdsey group had been transferred to Headquarters, Fighter Command, Royal Air Force, its activities widened, and tactics and policy began to be studied as well. Other sections became established, and the story of the development and success of war-time military operational research is now very familiar —perhaps excessively so—and all who will can read it.

Since the end of the war, the similarity of some of these military activities with some work previously done, and being done, in industry—work such as consumer research, rationalization of organizations, studies of production processes, work studies, and so on—has led to an extension of the use of the term operational

* Director of Operational Research, Admiralty. This paper is the substance of a talk given at the Institute of Engineering Production at the University of Birmingham in December, 1953.

research. This extension has, perhaps, created difficulties in the general understanding of the nature of the subject.

Military operational research is not new. It did not suddenly begin at Bawdsey in 1937, even though the phrase did. One is constantly struck, in reading military history, by examples, consciously or unconsciously applied, which are just as striking as any recent ones.

I do not suppose that the activities which are now described as operational research are in any sense new in industry. I am sure that those businesses which have emerged to success, must, at some time or other, in some form or other, have used some operational research, intuitive or calculated, consciously or unconsciously.

But what *is* new about operational research, I think, is its importance. Its new necessity in the military field arises from the rate of change of military equipment, of weapons, and of situations. Its importance will grow because the rate of change is not only fast but it has quite an appreciable acceleration. Warfare becomes more costly and now, more than ever, must we think before we act. I suggest that this is also true of industry. Perhaps, it is particularly true of British industry.

Even a hundred years ago, there was plenty of time to allow the practical men of war, of business, or of affairs their years and decades of trial and error. Now, there is not the time to let this process work itself out. Furthermore, some of the issues are now so big that, to quote, the trial and error procedure turns out to be one of "trial and catastrophe".

I believe that we who practise operational research must not yet claim to have discovered something new, but that we can say that we are consciously travelling a path that has been trodden before, in full realization of the growing importance of the work, and in the belief that, by the conscious application of scientific method, we can be right more often than those who do not use our methods. Time will show whether our claims and beliefs are right.

What is it that we do? In the military field, it is possible, somewhat artificially, to divide up into four classes the work which has been done and is being done under the name of operational research. They are:

(i) the determination of the operational effectiveness of weapons and equipments;

(ii) the analysis of the results of operations or (in peace) of exercises to determine the effectiveness of tactics, the influence of weapons on tactics and of tactics on weapons;

(iii) the prediction of the results of future operations;

(iv) the analysis of the efficiency of organizations or of methods.

VOL.
5
1954

The one thing that all these have in common, I think, is that they are concerned with the effect of changes. One seeks to study the organization, method, process, situation or equipment in the sense of estimating the effects of change, or to derive an optimum outcome, however defined. One tries to forecast changes in the output, or to estimate the new conditions of stability, or the new probabilities of success, which arise from a practically or theoretically applied change in the conditions.

Therefore I think that operational research is best viewed as the study of the dynamics of situations, of processes, of organizations, or of methods. This seems to me to describe, as well as I can formulate it at present, the content of operational research; and I think that, as in any other branch of science, the pursuit of knowledge in these matters for its own sake is entirely justifiable. The application of operational research, to executives, or anybody else, seems to me to be an entirely separate subject, and should not confuse the statement of the content of the work: but of course I would agree that, in a realistic world, proper application of the results of operational research at the proper level is really the aim.

If these views are accepted, we begin to see what sort of a man we need to do the work. We require, in the ideal, an individual who has:

(a) the conceptual ability to see the matter as a functional whole and to see what wants analysing;

(b) the expertise to do the necessary functional, dynamical or statistical analyses;

(c) the ability to do things the simple way;

(d) the ability to present the answer in a straightforward and understandable statement.

Who is this man but a good scientist?—or one who would have been a good scientist if he had been trained as such?

An essential quality of an operational researcher is a flair for the job, and a liking for it. I have yet found no way to induce this flair by education in the formal sense, although I have seen that if it is there at all, but latent, it may be brought out by clear leadership. This is the "art" side of the work, and, I think, when one examines it, it is really no different to a similar situation in normal scientific research. The flair does, however, need backing up in the same individual by a scientific approach to the finding of a solution, and it is this that we could do something about by training.

The general philosophical basis of operational research is, to my mind, still being built. We have not yet arrived at a stage where there are enough successes, enough failures, enough general data about

VOL
5
195

the subject itself to be able to formulate its own generalizations. Thus, I do not believe that a formal training *ab initio* for operational research can yet be devised, or, indeed, that it is justifiable. The normal training of a scientist is good enough, and he who wishes to enter into the work will find it easy enough to acquire any specific tools that he wants. The particular expertise required varies, of course, with the problem. The needs generally include some mathematical ability, to the stage of fairly elementary partial differential equations, analysis of variables, probability theory, statistical theory, information theory, cybernetics, and so on, and, never to be forgotten, skill in the design of experiments. Operational research should be as experimental as it is analytical. These specific abilities are the means to the end; they are not the end in themselves.

Not all good scientists make good operational research workers. Good mathematicians, I am told, seek exact solutions, and tend to approximate the problem in order to get one. In operational research, we always seek an approximate solution to the real problem. Operational research, with its inevitable compromises, is much more like an applied science, and may not suit those whose thoughts turn to purer paths. The actual work is largely a question of artifice and resource in the analysis and manipulation of the problems.

In the choice of operational research workers, it is obviously economical to start with men of graduate or equivalent standard; that is, people who are largely trained so far as the necessary expertise is concerned. In the operational research departments of the defence services of the United Kingdom, the normal practice is to put such a man into a current piece of work under experienced leadership, and let him learn on the job. He learns by precept and example, and by reading through past work. He is tested by results.

What is wanted now, I think, is not provision of intensive specialized training in operational research, but the creation of conditions that reveal the possibilities of operational research to those with the right aptitudes and talents, and that reveal the possibilities of its application to those who need its results. In doing this, it will be very important to proceed in the order in which I have written these two things, for nothing but disaster can follow from a situation in which an overselling of the possibilities of application leads to a demand for operational researchers which outstrips the supply of competent and experienced men.

I am grateful to the Admiralty for permission to publish this paper.

VOL.
5
1954

A SURVEY OF OPERATIONS RESEARCH IN THE U.S.A.

by

ELLIS JOHNSON*

OPERATIONS RESEARCH is expanding very rapidly in the United States industry at the present time. While the beginning of this high rate of expansion might be related to the formation of the Operations Research Society of America, which had the stamp of approval from the National Research Council and, so to speak, made the subject respectable, there is a growing feeling among operations research practitioners in the United States that the subject stems as much from the work of Taylor and others in the nineteenth century as from the war-time experiences to which it is more usually related. Between 1920 and 1940 it is probable that between 10 and 30 analysts were working in fields which would now be called operations research an example of which is the work of Levinson at Bamberger's.

It will be convenient, however, to discuss the growth of operations research in the United States since it has been formally known by this name.

GROWTH OF MILITARY OPERATIONS RESEARCH

Military operations research began in 1941–2 and by 1945 there were some 400 analysts. There was an immediate post-war fall followed by a rapid revival, established by the Rand Corporation in 1946, followed by the Weapons System Evaluation Group (WSEG) and the Army in 1948. Now, in 1954, there are about 800 analysts in U.S. military establishments distributed in the approximate proportion of WSEG 1, Navy 2, Army 10, Air Force 20. It seems probable that the peace-time ceiling has very nearly been reached; it may be about 1,000. However, growth is likely to be primarily towards the technical services and towards the combat development groups.

THE OPERATIONS RESEARCH SOCIETY OF AMERICA

O.R.S.A. in March, 1954, had 870 members, and the membership was growing at a rate of 40 members per month. The rate has been linear for the two years of its existence. Since only about one-third of the membership consists of people engaged in military operations research (the remaining two-thirds being, of course,

* Director, Operations Research Office, Johns Hopkins University, U.S.A. This paper is an abridgement of an address to the Operational Research Society in London, on 5 April 1954.

industrial) it is clear that only a fraction of military analysts eligible are members. The proportion of industrial analysts who are members of O.R.S.A. may be rather larger. A reasonable estimate would appear to be that there are about 1,500 analysts now in the United States, about one-half of whom are engaged in industry. About one-third of the total are invested with the title of operations research workers and the remainder are engaged in it under different names.

BREAKDOWN OF O.R.S.A. MEMBERSHIP AT 1 JULY, 1953

(a) By Industry Group

Type of Company	*No. of Companies*	*No. of Individual Membership*
Aircraft producers	11	36
Assoc. mfrs.	4	6
Electric mfg. cos.	9	16
Insurance cos.	8	12
Chemical cos.	7	10
Oil cos.	5	8
Utilities	4	11
Steel, copper and wire prod.	4	4
Department stores	3	3
Calculating machinery cos.	3	9
Automotive mfg. cos.	3	4
Miscellaneous mfg. cos.	31	40
Miscellaneous cos.	8	9
	100	168

(b) By Other Groups

	No. of Individual Members
Universities	91
O.R.O.	74
Military personnel	48
Consulting firms	47
O.E.G.	41
Research laboratories	40
O.A.D.	30
Rand Corporation	20
Individual consultants	15
Foreign members	14
Other government agencies	10
S.R.I.	7
W.S.E.G.	5
U.S. Naval Post-Graduate School	3
Miscellaneous	33
	478

The rate of increase of analysts in industry is probably between 20 and 100 per month and this high rate has been maintained for the

VOL.
5
1954

last two years. The number of formal operations research groups in industry has in fact doubled during the last twelve months. There are now at least 18 companies with formally established operations research groups and a number of other companies are doing operations research under different names. In addition, there are at least 13 firms employed by industry to carry out operations research.

An indication of the current industrial interest in operations research is that an article by Magee in the *Harvard Business Review*, which has a circulation of 38,000, sold no less than 12,000 reprints.

Also of interest may be an extract from a report of a group project under General Georges F. Doriot at Harvard University Graduate School of Business Administration in February, 1953. Letters and questionnaires about operations research were sent to the Presidents of 466 selected outstanding companies in the United States. Some 143 companies (31·8 per cent) replied and of these 134 answered the questions. The results may be summarized as follows:

- Total answering questions, 134
  - Unfamiliar with O.R., 88 (65·7%)
    - Expressed no curiosity, 21
    - Asked for further definition, 18
    - Interested in further information, 49 (36·8%)
  - Using or familiar with O.R., 46 (34·3%)
    - Actively using O.R., 9 (6·7%)
    - Considering or expecting to use O.R., 22 (16·4%)
    - Not using O.R., 11
    - Had used O.R. and would do so again, 2
    - Using O.R. in one area to determine further usefulness, 2

Among the 46 companies using or familiar with operations research were a flour manufacturer, department store, chemical producer, a medical and drug supply manufacturer, rubber company, petroleum company, producer of radio, television and other electronic equipment, maker of hydraulic shock absorbers, plastics manufacturer, meat-packing company, mail order company, optical company.

About one-sixth of the companies covered by the Doriot survey are potential customers for operations research. It can be estimated that two-thirds of the companies in the whole of the United States have little or no knowledge of operations research.

At the present time 65 per cent of the industrial O.R. in the United States is carried out in six fields:

Aircraft industries	Instrument manufacturing
Electrical machinery	Machinery manufacturing
Chemicals	Petroleum

VOL
5
195

It is a significant fact that these groups also account for about 75 per cent of the industrial research and development expenditure in the country. Other industries or groups represented in O.R.S.A. are department stores, calculating machinery, insurance, metals, utilities, and consulting firms.

The Scope of Operations Research

There are now about 50 different kinds of companies doing operations research with their own groups or through outside contract. Within these companies the range of work covers:

Sales and advertising	Crop scheduling in agriculture
Operations costs and standards	Airline and RR systems
Inventions	Accidents
Warehousing and transport	Government
Production	Schools and community

This scope is likely to increase, especially in view of the predominant American attitude of "let's take a try". Possible future fields of enquiry are school population, to put 10-year forecasts of requirements on a rational basis as has been done at Manchester, Mass.; traffic investigations such as those on toll booths carried out by the New York Port Authority who have brought their forecasts of operation to a high state of accuracy by putting them on a rational basis; disaster plans, as for methods of coping with atomic attack, considering a community plan as a whole, but taking in the family as a basis. A multitude of other promising fields of operations research in government departments comes to mind.

Cost of Operations Research

The annual cost of operations research in the United States appears to be about 20 thousand dollars per man. The salary of a "new caught" Ph.D. is 4 to 6 thousand dollars and that of the top leaders is 15 to 30 thousand dollars.

United States industry on an average spends 2 per cent of its gross sales on research and development. A company with only 500 employees might devote to research and development as much as 7 per cent of its gross sales while a company employing 10,000 or more might devote 2 per cent. Variation in any industry is over a range of seven times. A sample has shown that in organizations using operations research the ratio of O.R. expenditure to research and development is 0·05, ranging from 0·01 to 0·1. On this basis a company with 500 employees might now employ an O.R. group of

VOL.
5
1954

one or two analysts which might in the long-range future rise to as many as 5 analysts. The 10,000-employee company might now employ an operations research group of 10 analysts and in the future might double it. These increases are based on an assumption that the ratio of operations research to research and development might rise to one-seventh.

The present market, based on the Doriot survey, can be estimated at about 750 analysts. The present estimated rate of growth (500 to 1,000 analysts per year) is compatible with this. The ultimate market, if all companies using research and development adopted operational research at a 1 to 10 ratio, would be of the order of 10,000 analysts.

It seems reasonable to conclude that United States operations research will continue to grow rapidly and it is a matter for serious thought that this rate is likely to exceed our ability to train new analysts. This carries the obvious danger of degeneration in the quality and standards of operations research, and it might be said that this degeneration is already occurring.

Education for Operations Research

Twelve universities are giving seminars in operations research in the United States. These are:

Case Institute of Technology—open to public,
Johns Hopkins—open to public,
M.I.T.,
Columbia University,
U.S. Naval Post-Graduate School,
University of California at Los Angeles,
Penn State College,
Illinois Institute of Technology,
Stevens Institute of Technology,
Wright Field Air Development Center,
Tufts College,
American University.

Most of these are in the nature of surveys of the field and only recently have studies for doctorates been undertaken; the first of these is soon to be awarded at the M.I.T. The Case Institute of Technology will admit from 5 to 10 doctorate candidates this coming year.

There is a tremendous enthusiasm for one- or two-day courses which occur at the rate of 2 or 4 per month. The following list shows the large variety of industries and colleges which have organized

such conferences. In those organized by colleges and universities the local industries usually participate.

Railway Systems and Procedures Assocn.,
Society for Advancement of Management,
Columbia—Conference on Industrial Research,
Michigan State College—Industrial Engineering Inst.,
National Convention of A.I.I.E.—Louisville,
Washington Section of A.I.I.E.,
Wisconsin Engineering Institutes,
Case Institute of Technology,
Johns Hopkins University,
University of Minnesota,
Institute of Radio Engineers,
San Francisco Society for Advancement of Management.

In spite of these educational efforts it does not seem possible for sound training to keep pace with the present rate of growth of operations research. In many cases existing activities are re-labelled "operations research" and many charlatans enter the field by this means. There is a great need to face the mounting prospect of expansion of a necessary profession by meeting the challenge of holding the standards of training at the universities and elsewhere, and of maintaining the standards of membership of the Operations Research Society of America.

ABSTRACTS AND REVIEWS

Productivity Studies as a Science.

CHARLES F. GOODEVE (B.I.S.R.A.)

J. Inst. Production Engineers, September, 1953; **32** (9), 387.

Productivity is defined and its relationship to costs is discussed. As an example an analysis is given of the cost of making one ton of pig iron, the cost component being derived as the sum of the products of the prices of each resource multiplied by the resource consumption rate (the reciprocal of the productivity). Consumption rates are related only to the equipment, the process, and the organization and they are therefore more accurate than costs for comparisons or for predictions of future trends.

The make-up of consumption rates is discussed and particular attention is directed towards the importance of size of plant and the measurement of plant capacity.

VOL.
5
1954

The Human Factor in Relation to Operations and Machinery.
J. M. VAN DE WETERING.
Tydschrift von Efficientie Bedrijfsorganisatie, 1954; **9** (4), 100–105.

The author draws attention to some results of German research undertaken to obtain quantitative information on human performance in relation to working conditions. Modern design of machinery does not, or only to an insufficient degree, take into account the limitations to which the human body is subject. Since equipment and operator form a unit, efficiency is adversely affected.

Comparative Economics of Open-hearth and Electric Furnaces for Production of Low-Carbon Steel.
S. L. CASE, D. D. MOORE, C. E. SIMS, and R. J. LUND (Battelle Memorial Institute).
Bitumious Coal Research, Inc., July, 1953; 77 *pp.*; $10·00.

The Electric Furnace Survey Group, comprising fourteen electric utility companies and Bitumious Coal Research, Inc., sponsored this comprehensive study to appraise the competitive prospects of the electric process for steels which have hitherto been produced on a large scale mainly in open-hearth furnaces.

Costs and technical data were obtained from operators of modern plants of both types and from plant manufacturers; the figures quoted are "representative" of 1952 costs, and comparisons were made throughout for three sizes of plant—250,000, 500,000 and 1,000,000 tons per year—operating two different practices—cold metal and 50–50 hot metal/cold scrap.

Capital costs of electric furnace plants were found to be only 60 per cent of the costs of open-hearth plants of similar size, but against this electric furnaces have the higher operating costs. The cost of the metallic charge contributes about 75 per cent. of the total cost of production, and the authors argue that at the prices ruling in the U.S. for scrap and pig-iron an electric furnace on cold metal practice can melt cheaper materials; the electric furnace also gives a higher metallic yield.

When these advantages and disadvantages are combined it is concluded that with cold metal practice the electric furnace can produce steel more cheaply than the open-hearth, but that with hot metal practice the reverse is the case. Calculations of rates of return on capital are put forward to show that even with hot metal practice the electric furnace under certain conditions compares favourably with the open-hearth.

The paper does not pursue the comparison in conditions of less-than-capacity working, and no attempt is made to assign a

value to a number of less tangible differences in performance of the two processes. All costs quoted are, however, accompanied by statements of operating data and rates of consumption, and the reader can easily adjust the costs to take into account different prices and conditions. A large section of the paper discusses in detail the present trends of fuel and material costs, and there is a full description of the steelmaking practices involved.

D. G. O.

ROAD RESEARCH

The following six summaries refer to a selection of the work undertaken by the Road Research Laboratory. *They are reproduced by permission of the Director of Road Research, Road Research Laboratory, Harmondsworth, Middlesex, to whom anyone interested should refer for further information.*

The Effect of Rainfall on the Number of Road Casualties in Great Britain.

RN/1853/PCJ.

Data collected in Great Britain since 1945 have been analysed to see whether road casualties vary with rainfall. It was found that when the difference between the rainfall in a winter month and that in the previous month was above average, then the corresponding ratio of casualties was also above average. Rain in winter tends to increase casualties. They increase by about 1·5 per cent in November, and nearly 3·5 per cent in January for each inch of rain. In summer, however, increased rain probably tends to reduce casualties.

Much of the variation in the ratios of casualties in consecutive months cannot be accounted for in this way by variations in rainfall. Some at least of this is probably due to the as yet unmeasured effects of other major weather factors, e.g., snow, fog.

Some Factors Affecting Visibility from a Driver's Seat and their Effect on Road Safety.

RN/1957/RJS.

Investigations were made of the effect on road accidents of a number of factors that affect the vision of a driver. It was found that:

(1) During the war, when a blackout was in force, fatal accidents to pedestrians were twice as frequent on moonless as on moonlit nights.

(2) When related to the mileage travelled, those collisions between a

VOL.
5
1954

vehicle and a fixed object that involve personal injury occur 40 per cent more frequently during dark than during light hours.

(3) The effect of "summer-time" is to double the number of adult pedestrian casualties in the hours which change from light to dark and to halve the number in the hours which change from dark to light.

(4) Wet roads are associated with an increase of pedestrian casualties amounting to nearly 40 per cent in daylight and 130 per cent at night. These increases are probably due largely to the effects of rain on visibility.

(5) At night, vehicles are less likely to be hit in the rear if they have good rear lights.

(6) Compliance with the regulations concerning pedestrian crossings is better where the crossings are made more conspicuous to the driver.

(7) The accident rate per vehicle mile is less on roads with good sight distances than on other roads.

Some results obtained in the U.S.A. suggest that drivers with eye defects tend to be involved in more accidents than drivers whose sight is satisfactory.

The Effect on Pedestrians and Vehicles of a New Zebra Crossing in Slough High Street.
RN/1983/SJO.

A new pedestrian crossing was put down in Slough High Street in February, 1953.

"Before" and "After" surveys of a 125-yard section of the street centred on the crossing showed that, of all pedestrians crossing the road in this section, 8 per cent crossed on the site of the crossing before and 52 per cent after it was laid. Although the crossing attracted mainly the pedestrians who would otherwise have crossed within 30 yards on either side of it, some people were attracted from distances up to 60 yards.

During the second survey about 6 per cent of all vehicles passing the crossing gave way to pedestrians using it. A similar percentage was found for vehicles giving way at an existing crossing in Slough.

Vehicle Inspection.
RN/2000/GG.

The note summarizes information on vehicle inspection. Facts concerning the state of vehicles at present on roads in Great Britain

are first given, and are followed by evidence of the effect of vehicle inspection in the United States of America.

With the co-operation of the police, the braking performance of a random sample of vehicles was measured. The results showed that an appreciable proportion of the vehicles using the roads have a low overall brake performance; for example, 1 per cent did not attain a maximum deceleration of 0·2 *g*, and 12 per cent did not attain 0·4 *g*. The stopping distances corresponding to these decelerations are approximately three times and twice those given in the *Highway Code*.

In another series of tests it was found that about 30 per cent of all cars tested had at least one wheel with practically no braking. In 12 per cent of cases, there was no braking on at least one rear wheel, and in 6 per cent no braking on at least one front wheel. Experiments showed that rear wheel unbalance was comparatively unimportant but that complete unbalance of the front wheels could cause the car to turn completely round when making an emergency stop even from a comparatively low speed.

Surveys of the state of vehicle headlights reveal that a high proportion are badly aimed and that many have deteriorated badly. Not more than 10 per cent were aimed within half a degree of the correct direction, while about 50 per cent were more than 3 degrees out (3 degrees is the angle between the main and dipped beams). One-sixth of the headlights examined had less than one-tenth of their original intensity.

Statistical and experimental evidence indicates that rear lamps should have an intensity of at least one-quarter of a candela. Surveys made in London and the Midlands revealed that 75 per cent of vehicles other than cycles, and 98 per cent of cycles, have rear lights less bright than this; 80 per cent of cycles and 60 per cent of commercial vehicles have rear lights less than one-tenth of this brightness.

The following common faults in the use of rear-view mirrors were found:

(1) In 10 to 20 per cent of commercial vehicles, there is an obstruction between the driver's eye and the mirror.
(2) Two-thirds of commercial vehicles have the mirror placed too near to the body work to give an adequate field of view.
(3) Less than one-half of the heavy commercial vehicles are fitted with nearside mirrors, although these vehicles are particularly liable to have left-turning accidents in built-up areas.

Evidence as to the effect of vehicle inspection comes mainly from the United States of America. Some measurements by the author showed that dazzle from headlights was less in states having inspection than in those without inspection. The evidence concerning

VOL.
5
1954

accidents is inconclusive. Compared with other states, those in which vehicles are inspected have accident rates which, on the average, are 20 per cent lower, but it may be argued that other safety measures also contribute to the reduced figures in these states. If the figures are accepted, however, and assumed to apply to Britain, and if costs are also assumed to be similar in the two countries, vehicle inspection would be economically justifiable, even if the resulting improvement were only $2\frac{1}{2}$ per cent instead of 20 per cent.

Traffic Congestion in Central London in Wet Weather.
RN/2010/JKT.

When buses in Central London are unable to keep time because of delays, arrangements are made so that some buses do not run the full route, but turn round at a predetermined point short of the terminus. The revised destination is exhibited on the blinds. The difference between the mileage of the schedule and that actually run is termed "lost mileage"; it may be regarded as an index of traffic congestion. Values of this mileage for weekdays in 1951 have been kindly made available by the London Transport Executive.

The mileage "lost" per day varies greatly and is affected by holidays, special events and rainfall. Eliminating days thought to be affected by special conditions, it appears that "lost mileage" increases with increasing rainfall and with the proportion of the day during which the road is wet.

It is inferred that rain increases traffic congestion in London and this conclusion is supported by observations on a number of main roads in Central London which suggest that speeds may be about 10 per cent slower on wet days.

Effect on Accidents of Striping Pedestrian Crossings. Review at end of April, 1953.
RN/2023/FG.

Two detailed investigations had previously been made into the effects of the pedestrian crossing regulations of October, 1951, and are recorded in report RN/1807.

The present note briefly reviews the position; it compares the ten-months period of 1951 before the regulations came into force with the corresponding period of 1952. It is shown that in boroughs and urban districts of England and Wales, i.e., where the bulk of the pedestrian crossings are to be found, pedestrian casualties were 8 per cent less in the 1952 period, whereas in rural districts they were only 2 per cent less. In Northern Ireland, where there are no such crossings, there was a 2 per cent increase in pedestrian casualties.

The figures for individual towns showed that those which had pedestrian crossings before the regulations derived more benefit from the changes than did other towns and that no abnormal effect occurred in those places from which, as the result of the changes, the greatest number of existing crossings were removed.

Casualties fell most on Class I and trunk roads and during the hours of darkness: a possible reason for the latter is the fact that the striped crossings are easier to see at night than the original beacons and studs.

During the first four months of 1953, pedestrian casualties increased by 5 per cent over 1952, but were still 5 per cent lower than in 1951.

It is concluded that, while other factors may have been operating, the reduction in casualties from 1951 to 1952 has been due, at least in part, to the pedestrian crossing regulations, and the benefits are probably continuing.

LARGE NUMBER RESEARCH AND FREQUENCY ANALYSIS

by

MANFRED KNAYER

IN 1922, a young engineer, Karl Daeves in Duesseldorf, began to survey steel industry products from a new point of view. He investigated the properties and the differences of the properties of large numbers and populations of pieces, as well as the properties of individual pieces. This new approach he called *Grosszahl-Forschung*—large number research, wide coverage research. It, together with frequency analysis and related methods developed by Daeves, Beckel and others, has proved useful in many fields of science and industry, including agriculture, biology and the medical sciences. As might be expected, however, the most fruitful field of application has been industrial, covering steel plants, textile, glass, food and consumer goods industries. In 1951 a meeting and discussion on Grosszahl-Forschung was held in Duesseldorf, where some 30 experts presented papers on the practical applications of large number research and frequency analysis.

The new approach prefers graphical to numerical statistical methods for the presentation, analysis and evaluation of data. Two means have greatly facilitated the application of graphical methods:

Probability Paper. When drawn on probability paper the bell-shaped normal distribution curve is converted into a smoother curve

VOL.
5
1954

resembling a hyperbola. More important in use is the cumulative frequency curve and the cumulative frequency percentage curve. Each point on such a curve clearly indicates the percentage of the total items lower than the indicated value. This curve when drawn for a normal distribution on probability paper is converted into a straight line, which considerably simplifies the drawing and judging of distributions. If the distribution differs from normal, the cumulative frequency percentage curve correspondingly differs from the straight line. The differences make it possible to judge whether the population consists of two or three entangled populations. Another advantage of the cumulative frequency percentage curve is that it is independent from the width of the class intervals.

Logarithm Scales. Skewness may sometimes be eliminated by changing the kind of class mark or class mark scale. A logarithmic scale often converts nonsymmetrical distributions into perfect normal curves.

The use of graphical methods has been justified by many applications and results. It is claimed that they permit frequency analysis to be applied even by engineers and managers with little knowledge of statistical methods. Together with analysis, graphical methods help to improve the quality of products by revealing the factors of major influence, even before the real reasons for dependence are known. Forecasting becomes possible in certain conditions of large number research, when one evolution of certain phenomena can be observed. The value of the technique in getting results when only a small number of observations is available has become increasingly apparent. In some cases 16 measurements or observations formed a sufficient basis for evaluation of a situation.

Schwachstellenforschung, literally, weak point research, or failure research, is a special application of large number research. By counting the failures of products during use and by classifying them properly, the weak points in material or design may be detected. For instance, the strength of miners' shovels and the durability of pneumatic tools were considerably improved by investigating and counting either the breakages or the consumption of spare parts and by modifying and redesigning such tools. Similar results were obtained in the watch industry.

References

Daeves and Beckel, Grosszahlforschung und Haeufigkeitsanalyse, *Verlag Chemie, Weinheim an der Bergstrasse*, 1948.

Rationalisierung durch Grosszahlforschung. (Report on the meeting in Duesseldorf in 1951. Edited by Dr.-Ing. Karl Daeves.) *Verlag Stahleisen, Duesseldorf*, 1952.

RECENT COURSES

Institute for Engineering Production, Birmingham University

The second two-week course on the *Use of Operational Research in Increasing Productivity* was held from 20 April to 1 May, 1954. The course followed similar lines to the first (reported in *O.R.Q.* Dec. 1953, **4** (4), 77–80) but advantage was taken of the previous experience to provide a better integrated programme.

In addition to English representatives, the second course was attended by 15 visitors from O.E.E.C. countries including Denmark, France, Germany, Greece, Italy, Netherlands, Norway and Sweden.

Linear Programming

Some fifty organizations, industries and government departments, were represented at a London conference on 4 May, 1954, organized by Ferranti Ltd., to discuss Linear Programming. The news that a computer-programme now exists for the "Simplex" procedure on the Manchester University computer may encourage new applications.

The following papers were presented: *Linear Programming of an Air Lift* by Dr. S. Vajda (A.R.L.); *A Problem in Transportation* and *A Dynamic Programming Problem* by Mrs. A. Land and Dr. G. Morton (L.S.E.); *Computation of a Diet Problem on the Manchester Computer* by J. A. C. Brown (University of Cambridge); *Linear Programming by the method of Loading Variables* by E. N. L. Beale (A.R.L.); *Some Experiences with the Simplex Method on the Manchester Computer* by Dr. D. G. Prinz (Ferranti Ltd.).

Hazards of tackling problems with very large numbers of variables, and the desirability of obtaining near-optimum solutions for some purposes were among the topics of the discussions between the papers. A report of the proceedings, including only synopses of the papers, will soon be obtainable from Ferranti Ltd., 36 Kingsway, W.C.1, who will welcome enquiries about the conference and any information about new applications.

Madrid University

A course on the *Applications of Operational Research* was held at Madrid University, from 26 April to 1 May last.

VOL.
5
1954

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OPERATIONAL RESEARCH QUARTERLY

VOL. 5 NO. 3 SEPTEMBER 1954

Contents

Who plans what? 57
Inspection——E. D. VAN REST 60
Assessing Priorities for Technical Effort——G. H. L. ANDREW . 67
Criminology and Prediction——L. T. WILKINS 80
Abstracts and Reviews 87
O. R. and Accounting 92

EDITORIAL NOTES

WHO PLANS WHAT? AND WHY? AND HOW?

UNDER this title in the *National News-Letter* of 5 August, the editor, Sir Stephen King-Hall, hits a large number of nails robustly on the head. He takes as a "text" the recent House of Commons debate on a bill authorizing the nationalized electricity and gas boards to borrow, subject to the approval of the Chancellor, sums of £700 million and £300 million respectively. This leads him to consider the whole subject of capital investment and the approportionment of national resources.

His argument runs somewhat as follows. The state now plays a very large part in determining the course of national investment. It intervenes thus largely in economic life partly "as a result of a normal evolutionary process in a society of growing complexity, and partly because this process has been accelerated by two world wars". In such an involved society "problems become so large in physical terms that only a central authority has *or appears to have* the necessary resources with which to solve them" (original italics). But state investment obscures the need for an economic return which disciplines private investors, and lays upon those responsible for decisions the duty of enunciating the other objective criteria which they accept, in any particular case, in place of it.

In the debate on the Gas and Electricity (Borrowing Powers) Bill such an enunciation was conspicuously absent. "The question came up in debate", writes Sir Stephen, "as to why earlier in the year the Government had announced that a proposal to raise a loan

of £500 million for road development was rejected on the grounds that it would upset the money market and be dangerously inflationary, whilst a few weeks later Parliament was being told that it was in the national interest to allow the state-owned gas and electricity undertakings to raise and spend £900 million on new capital investment. To this pertinent question the Minister prudently refrained from attempting to make a reply.

"It is, however, a question that will have to be answered sooner or later. Who is it who decides that it is in the national interest to put our savings into gas and electricity and not into roads? What were the arguments for and against? No one knows. Moreover, the test as to whether the right decision has been made will never operate because the nationalized boards can never go bankrupt. Also, since they obtain money from the state without having to run the commercial risk of failure, what incentive other than pride in their work have these boards to be efficient?"

There is no quick, easy answer to the fundamental question posed by the *National News-letter*. Whatever the truth, the article goes on to say, "It is certain that we are all in a state of abysmal ignorance as to how we are trying to replace the laws of supply and demand by the new techniques of state planning. We do not know who are doing it, nor why they do what they do". The article goes on to suggest that a start on the necessary exploration might be made in the nationalized industries. An attempt should be made to "find out on what basis, if any, the nationalized industries are given priority and complete security for capital investment. At present all this is concealed from Parliament. Is it because Parliament as a body is not fitted to decide these matters, such as whether roads or gas are the more important from the national point of view? If that is the answer, then there is something wrong with Parliament, which is supposed to be the sovereign body representing the people whose individual needs add up to the national interest".

Operational research workers in particular will welcome this suggestion of examining a going concern, and it is hard to see how a sufficiently objective assessment could be made by other than scientific methods. Many of the nationalized bodies have their own effective operational research sections, as the columns of this Quarterly have shown from time to time. But the very great scope still open to operational research is indicated by the argument summarized above. The sights of operational research should eventually be set at making available and comprehensible the objective, rational evidence on which economic decisions at board and ultimately national level can be based. The scientist is often

VOL.
5
1954

thought of as the ultimate expert—an enemy to freedom because beyond the real control of his uncomprehending contemporaries, a negation of the democratic safeguard of inexpert control over experts. But in Sir Stephen King-Hall's plea the wheel comes full circle. The scientist is called in to assess and make clear the forces which democracy itself is failing to control. The expert is to provide the evidence on which the inexperts are to base their decisions.

This is, in fact, not a bad formula. Probably in to-day's conditions it is the best we can do. At any rate it is a rational approach to comprehension and control of some of the forces that threaten freedom.

"Some parliamentarians (with safe seats)", writes Sir Stephen, "should break out of the cocoon of party discipline which is thickening round them and reassert the sovereignty of Parliament by probing into the question, 'Who plans what and why and how?' "

How right he is.

OPERATIONAL RESEARCH SOCIETY

THE following programme has been arranged for the 1954/55 session. All meetings except the December meeting will be held at the rooms of the Royal Society, London, at 5.0 p.m., preceded by tea at 4.45 p.m. The December meeting is this year's "open" meeting, and will begin at 2.30 p.m. The first meeting will be preceded by the Society's Annual General Meeting.

13 October: *Application of the Birth and Death Process to the Provision of Spare Machines*. Speaker: R. R. P. Jackson. Chairman: Dr. O. H. Wansbrough-Jones.

19 November: *The Use and Abuse of Analogy*. Speaker: Miss V. Daniel. Chairman: E. C. Williams.

20 December: *The Accuracy and Function of Time Study*. Speakers: Prof. T. U. Matthew and W. Rogers. Chairman: A. W. Swan.

18 January: *The Handling of Uncontrolled Operational Research Data*. Speaker: Not yet known. Chairman: R. T. Eddison.

15 February: *The Mechanization of Ticket Printing and Issue*. Speaker: R. J. Eaton. Chairman: Sir Charles Goodeve, F.R.S.

15 March: *Problems in the Coal Industry*. Speaker: Dr. R. Revans. Chairman: D. Hicks.

INSPECTION

Its Relation to Operational Research*

by

E. D. VAN REST

ALMOST all examples of operational research discussed by the Operational Research Society have been descriptions of work directed towards the improvement of a process or of an operation. Such work usually involves the study of the process as it is, the creation of a model which allows prediction to be made of the effect of changes in some of the conditions, and the testing of these effects in actual practice. The O.R. team then moves on to another problem.

Inspection differs from this kind of study in being an example of continuing operational research. The process studied has already been brought into a satisfactory state; inspection may have contributed towards this but its main function is to keep the process in a satisfactory state. Since most people are of the opinion that once a process has been specified and found satisfactory it ought to be perfectly easy to keep it running, it is not difficult to understand why the need for inspection is often overlooked or why it is regarded as a minor activity and one not worthy of much study. I have come to the opposite conclusion. Since becoming interested in inspection as a subject embracing other things than its statistical techniques, I have been surprised to find how much there is to say about it and how it falls naturally into the pattern of true operational research which I take to be, quite generally, the study of processes.

Although much of what I have to say is tied to ordinary manufacturing processes, I like to think that the ideas and principles apply to any process. Almost all that we do can be considered as a series of processes and as such liable to go wrong and therefore, in differing degrees, requiring inspection. I shall show later, too, that although inspection's main task is to keep the process running as it started, it has much to contribute to improvements of the process and it would be wrong, therefore, to confine its activities to the main task.

WHY INSPECTION IS NECESSARY

It has already been said that the need for inspection is often overlooked or the activity is regarded as a minor one; this is especially so in the smaller firms. It is probably true to say that the smaller the firm the less a separate inspection organization is needed until in the extreme one individual working alone must do his own inspection.

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* Paper given to the Operational Research Society on 16 April, 1954.

VOL.
5
1954

There we have the root reason why inspection is necessary at all. One man working alone, the craftsman, will inspect his own work; as soon as two are involved on the same task communication is necessary and the imperfection of communication leads to mistakes. Also, men and machines are both fallible and are sources of change in the process. That the need for inspection does not arise solely from lack of good intentions is suggested by the following extract from the 1951 report of the committee on Weights and Measures Legislation:—

"We discovered that its methods (for ensuring that the weights and weighing machines used in post offices throughout the country were maintained in an accurate condition) were open to objection and would not, if used by a private trader, have been regarded as adequate by any Inspector of Weights and Measures. We are convinced that no amount of integrity of purpose inside departments can afford a guarantee against the use of inadequate methods sufficient to enable the spur of independent inspection to be safely dispensed with."

In considering the need for inspection it is profitable to consider the parallel activities of costing and production progress. A manager should be concerned to know about his organization—what it costs and what it is doing. The information about what it is doing must necessarily be incomplete if it contains details of quantity only and not of quality, i.e. whether the product is the product that has been specified (and for which selling arrangements have been made).

On What Principles Ought Inspection to be Organized?

Unless there is agreement in any one organization on the principles on which inspection ought to work, it is unlikely to play its full part in the overall task of production of a specified article. The reason would be that there would not be agreement on the part it ought to play and the responsibility it ought to take in each part of the organization. A further reason in favour of a statement of principles is that one is enabled to check more easily whether a proposed set-up will be able to carry its required responsibilities. The need for agreement on principles is all the more urgent because there is not one unique set of principles. The one put forward here is, no doubt, only one of many such possible sets; all that is claimed is that it is a logically self-consistent set and would fit logically into the kind of manufacturing set-up met with at the present time.

The seven principles stated below are not meant to be exhaustive even for a self-consistent set; they are rather set up to enable the function of inspection to be presented as a consistent whole and

to enable the detailed functioning of inspection in any particular situation to be easily specified.

The principles are:

1. That every organizer owes it to himself to check the working of his organization.

2. That the checking function may be delegated and is often better delegated.

3. That the existence of the check does not remove any responsibility from other parts of the organization to produce exactly what was intended.

4. That since inspection, properly linked to the remainder of the organization, can be a form of control on quality and since the passing of information is an essential part of a control system, special attention must be paid to the provision of the links and the channels of communication.

5. That control is more effective on the *process* than on the finished product.

6. That there are many forces, other than the actions of the producer, which ought to exercise some control on quality (prospective customers' wishes, for example) and these should be channelled in some way to enable their influence to operate.

7. That inspection in all its fields of activity has a three-fold function to perform—that of accepting, of preventing, and of assuring.

Discussions of the Principles

1. This need holds at all levels from the operator checking his own work or his machine at intervals to the managing director or the Board checking each of their many plants. Provision should therefore be made at all levels; time, apparatus and perhaps assistance for the operator, reliable information for the board.

2. Delegation of the function is often more efficient especially where inspection involves specialized equipment. It is often desirable since the urge to complete a job on time may lead to relaxation of the quality requirement by the production man, but not if reminded by another person of any falling off. Delegation must involve a channel for information to be passed back to the person delegating his inspection function. The operator cannot control without the inspector's information; the manager must constantly be informed; no news is not good news.

3. If the inspection department is made responsible for quality it becomes only a sorting mechanism, any incentive to make correctly is removed from production and scrap would be expected to increase.

VOL.
5
1954

If production does not take responsibility for quality the greater part of its function is missing.

4. Unless information is passed there is no possibility of controlling the process. Rapid and accurate passage of information can help towards better control.

5. A form of control which obtains its information from the finished product must often act too late to be efficient in control. A watch on the process is often a more sensitive indicator of change. For example, a change in temperature of a stage of a heat treatment process would be more readily and quickly detected than a change in the properties of the product.

6. Unlike the positive actions of the producers the other influences which ought to control quality have to be sought out by field experiments, life tests, market research analysis of service reports, and their influence must deliberately be brought to bear (to act only when the sales begin to drop off is to act too late).

7. The inspector is the arbiter of what should be passed to the next stage, he accepts or rejects; he should, however, pass back the information he has gained from his inspection in order to control the process, to prevent any continued production of unsatisfactory articles. As the holder of a delegated function, he must pass information to the person whose inspection function he is carrying. He assures this person, as he also assures the user of the product.

The Organization Needed for Inspection

Inspection cannot, of course, exist by itself—it must be looked at as one function of a team. We cannot therefore study inspection in isolation nor without looking as a whole at the problem of producing a satisfactory article. To set up and maintain a manufacturing organization requires many sorts of activity; finance to buy the machines; staff welfare to recruit and "maintain" the staff; maintenance for the machines; design; methods planning; purchasing; inspection; sales. The problem of management is to organize a team exercising all these functions with the result that the desired article is produced.

There must be many ways of doing this, so rather than describe any one set-up it would be preferable to examine the principles just stated and to see how each leads to a requirement to be satisfied in the ideal organization.

1. The first, that everyone has an inspection function, calls for an inspection organization with representatives at every level of the chain of production responsibility, with communication channels both laterally and vertically.

2. Delegation of the inspection function is made wherever feasible and efficient, especially where otherwise there is likely to be a conflict of functions carried by one person.

3. The chain of responsibility running through the strictly production part of the organization must include responsibility for quality. Production must, therefore, be allowed some say in the direction of those who exercise its inspection function. This can be achieved by having the routine inspection done by production employees while the tools and the way it is done are the function of a "quality department" which has also other responsibilities.

4. The fact that the result of inspection is to be used to control the process means that provision must be made for the exercise of controlling forces, not only channels of communication but channels of influence: there must be means of ensuring that the correct changes in the process are made as soon as the need for them is demonstrated.

5. The exercise of the control on the process rather than on the product calls for an inspection organization with representatives at all levels intimately concerned with the process at all its stages.

6. The need to channel other outside influences on quality into the manufacturing organization requires a section constantly obtaining and analysing information from these outside sources in the light of information obtained from inspection within the organization.

7. In order to exert its function of acceptance, inspection must be in close touch at critical stages of design, purchase and manufactures; to prevent, it must make its analysed information available very rapidly; and to assure, it must analyse, condense and report.

It is possible, especially in the small firm, for all or some of these functions to be exercised by those who are also carrying others of the team functions already mentioned. If this is so care should be taken that there cannot arise any conflict of function in the same person as when both speed for schedule and slowness for quality are required of the production personnel. Similar conflicts, of finance and efficiency for example, are rarely allowed to be resolved by the one person: it is equally important that the quality controlling function should similarly not be subject to competition.

There will be many parts of the manufacturing organization in which the inspection function is needed and the separate sections called for will need to be linked into a vertical organization. More than some other functions of the manufacturer's team, the inspection function needs to permeate the whole: if at any stage this vertical chain tends to be broken, as for example when production deems it advisable to have its own inspection, then it is restored by inserting a

VOL.
5
1954

corresponding link with the function of simple checking on the activities of the part broken away.

The diagram (figure 1) indicates an organization with these properties. The intercommunications are the parts which are usually overlooked. Those must not be left to chance or to goodwill on the part of those concerned. Provision must be made and be kept in being not only for the passage of information at every stage but for action to be taken on that information for control purposes.

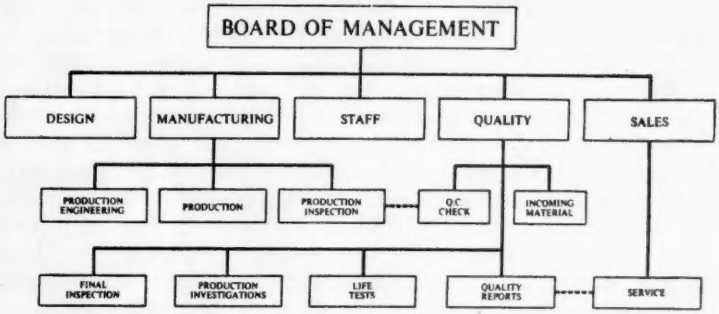

FIGURE 1. *The quality organization in relation to the other parts of the production team. (Reproduced from "Applied Statistics" by courtesy of the Editor and Publishers.)*

THE TECHNIQUES REQUIRED

The techniques required are dictated by the needs they serve. Rapid availability of information obtained at many stages of the process calls for either self-checking processes, automatic inspection or sample inspection. Total inspection is a last resort but often cannot be dispensed with especially at the last stage before the customer receives the article. Enough has been said to justify looking upon inspection as part, an indispensable part, of the manufacturing process. Economies on the manufacturing process are only justified so long as quality is maintained; equally economies in the inspection process are only justified so long as the information about quality and the power to control quality is maintained. These techniques, especially the first two which are peculiar to a process, require developing and this can be one of the functions of the inspection department.

Quite apart from the sampling methods usually associated with inspection, statistical methods are indispensable for the simple reason that inspection is dealing with variability.

The full exercise of the inspection function as just described meets all the definitions of operational research. It is the study of a stochastic process, of an operation, with a view to its stabilization. It takes "an active attitude towards the operation" with a view to its improvement. Since it is concerned with the outcome of the process it has no limitations to what it shall study—anything which could affect quality is within its province.

In pursuit of this idea we may note that while total inspection gives some (though not perfect) assurance that what the customer will get is correct this "total" is only a sample of what the process might at that time produce. Looked at as a sample it is probably unnecessarily large; unnecessarily large, that is, for giving the information needed to control the process so that the only excuse for total inspection is its assurance to the customer. The best assurance of perfect quality, however, is the knowledge that the process is continuing as it was on some previous occasion when known to be producing perfect quality. The value of thus controlling the process rather than merely examining the product has been argued above in principle (4); what is here remarked is the value of the *attitude* of regarding inspection as a means of control on the process. Such an attitude may lead one to adopt forms of inspection which only indirectly take the customer's view point yet because they control the process may be more efficient and ultimately lead to greater customer satisfaction.

Acknowledgement is made to the Chief Scientist, Ministry of Supply, for permission to publish this article.

Opportunity in the Coal Industry

The National Coal Board's Field Investigation Group is recruiting operational research workers interested in making a career in the coal industry. The work differs in some respects from classical operational research. The data required are rarely available in records and the Group has to collect them by underground studies at collieries. Conclusions are based, in the main, on approximate figures and refined techniques are not always required. The need of the coal industry for this type of work is great and is being increasingly recognised.

Salary scales range from £500 to £1,150 per annum, plus a London location allowance of about 8%. Scientists interested are invited to write to the Field Investigation Group, National Coal Board, 2 Grosvenor Place, London, S.W.1.

VOL.
5
1954

ASSESSING PRIORITIES FOR TECHNICAL EFFORT

by

G. H. L. ANDREW*

CAN quantitative analysis of a situation help us decide the best deployment of limited resources?

We shall examine the problem where the limited resources are technical and the field of deployment is an industrial company. The technical resources consist of expert staff, experimental equipment and allocated money; the deployment is between fundamental research, technical service to customers, factory "trouble-shooting", development of new processes and products, reduction of manufacturing costs and so on.

Take the familiar analogy between academic research and fishing. If the academic research worker appears more brilliant than the applied or industrial scientist it may be because he merely catches any fish that comes his way, whereas the industrial scientist has to catch one particular fish within a prescribed time. Now what we are examining in this paper is how to decide which of the many fishes is the best one to catch. What we need to know is firstly, how much we want each of the fish, and secondly, how difficult the catching is likely to be.

The planning of a company's technical strategy is frequently carried out by a committee with representatives of various and perhaps conflicting interests, such as production costs in opposition to the quality of the product. It is not suggested that such a committee can be replaced by an electronic computor; but can its subjective decisions be simplified and reduced in number, by the appropriate quantitative analysis of the situation?

THE IMPORTANCE OF THE SUBJECT

Much has been said about the need for more intensive exploitation of fundamental scientific knowledge in this country[1–5]. Many organizations and industries are already spending 1½ per cent or more of their turnover on research and development[6]. The implication is that still more should be spent, particularly on the application of scientific knowledge; but, in the immediate future at least, limitations are imposed by the lack of suitable scientists and technologists.

This paper suggests that in addition to seeking more scientific manpower we should make an increased effort to decide the best

* Based on a paper to the Operational Research Society, 10 November, 1953.

way of deploying the resources actually available now. We might well make better use of our total technical resources if we allocated, say, 2 per cent of them to finding the best deployment. We have not attempted in this paper to put forward a procedure applicable to all situations, and still less to propose a general formula for all decisions; what we have tried to do is to suggest some general lines of thought by means of particular examples.

A Practical Example

What do we mean by a quantitative analysis of the situation? Let us look at a practical problem of deciding between a number of development projects competing for limited technical resources, and let us consider whether the choice can be facilitated by analysing the complex mass of information into simple groups.

There were 12 different extrusion processes for making various kinds of nylon yarns; all of these processes needed developing separately from pilot to production-scale operation; the problem was to decide the order in which they should be taken. Differences between processes in costs of development and in profitability were of secondary importance and were neglected.

The main arguments in favour of the various processes could be grouped under the four headings shown in Table 1. Every project was rated (0–2) according to its priority under each of these headings. The ratings were subjective, except for those under "Extra Output". It will be seen that some processes acquired a priority in their own right, others did so because of the characteristics of their product.

The four headings were:

1. *Extra Output*. It was desirable to choose for development, processes which had an increased output so that they would allow the total production to be increased. The rating under this heading was defined as the number of extrusion machines which would be released by introducing the new processes. The figure was obtained objectively by multiplying the number of machines currently in use by the fraction of machine capacity released.*

There was a very urgent need for additional output equivalent to the capacity of one machine but the value of any more machines was much less.

2. *Quality*. Certain processes claimed priority because they gave a product of higher quality than the present one. The arguments put

* The possibility of separating two such factors which are then multiplied together may be quite general—one factor is "intensive", e.g. *Fraction of machine saved* or *Need for improvement* and the other is "extensive", e.g. *Number of machines operating* or *Magnitude of expected improvement*. One might therefore consider the dimensions of Priority as—Intensity of Need × Scale of Operation.

VOL.
5
1954

forward for processes under this heading differed in their relative emphasis on (a) the need for improvement in the existing product (rated 0–1) and (b) the magnitude of the expected improvement (rated 0–2). Hence, the rating was obtained by multiplying the two separate ratings to give the "value of the expected quality improvement over that of the current product".

3. *New Markets.* The processes were rated subjectively according to their ability to make yarns which would open up new markets or establish existing ones more firmly.

It was apparent that this rating was the most speculative of the four; it involved the estimating of both the size of potential markets and the chances of the hypothetical product capturing them.

4. *Evaluation of Equipment.* There was one particular item of process equipment the correctness of whose design could only be fully evaluated in the course of full-scale development of certain of the new processes. The equipment was common to all the projects but some projects provided a more thorough testing and were rated (0–2) accordingly.

TABLE I

PRIORITIES FOR DEVELOPING NEW PROCESSES

Reason for priority / Yarn type	I Output	II Quality	III New Markets	IV Testing Equipment
	Extrusion m/cs released i.e. no. of m/cs currently in use × fraction of m/c capacity released	Value of expected quality improvement over current yarn, i.e. quality improvement (0–2) × need for it (0–1)	Sales or commercial development need for new type of yarn (0–2)	Importance of proving equipment (0–2)
15 denier	0	2 (2 × 1)	0	0
20 ,,	0	0 (2 × 0)	0	0
30 ,,	2 (4 × ½)	0 (0 × 0)	0	1
40 ,,	*	*	1	1
45 ,,	4 (8 × ½)	0 (0 × ½)	0	1
60 ,,	2 (4 × ½)	1 (2 × ½)	0	1
70 ,,	*	*	2	1
100 ,,	*	*	1	1
150 ,,	*	*	1	1
205 ,,	1 (2 × ½)	2 (2 × 1)	1	2
210 ,,	1 (2 × ½)	1 (2 × ½)	0	2
250 ,,	0	0	1	0

* New product, hence no rating for saving of spinning machines or for quality improvement over existing yarn.

The immediate benefit of an analysis such as that in Table 1 is the clear presentation of the relevant data and opinions. But the crucial step in deciding which process to develop first, still remains —it is to determine the relative importances of the four columns. This step requires a directive on policy and cannot be created by any such table; on the other hand, the analysis helps to make it clear what exactly the policy directive must be about.

The directives given in the example we are examining were of the following form:

1. It is of overriding important to test the new equipment fully.

2. With this condition satisfied, the next most important requirement is that the output of at least one spinning machine should be released. It would be of some use to release a second machine but its value would be much less than that of the first; the value of releasing subsequent machines would be negligible. This meant that scores in column I exceeding one carried little extra weight.

3. When these conditions have been satisfied, equal weight should be given to "Quality" and to "New Markets".

These directives could be re-stated as:—

1. Select projects with rating of 2 in column IV.

2. From these, select those with rating of at least 1 in column I.

3. From these, select the project with the highest score for columns II and III combined.

From these considerations project number 10—i.e. 205 denier—is indicated as the best choice for the first process to be developed.

Advantages of the Analysis

What have we gained by analysing the problem in this way? Essentially the analysis has not made the decision but has facilitated it.

Firstly, tabulation has presented the relevant information in a form in which it can be readily assimilated and discussed.

Secondly, grouping the data into things-of-a-kind has permitted a straightforward rating even where it is necessarily subjective. In terms of a committee, the ratings in any particular column are within the province of one member who has a specific functional responsibility and they do not require discussion and compromise by the whole committee.

Thirdly, the analysis has narrowed down the field requiring the resolution of conflicting interests until it focused attention on

VOL.
5
1954

the clearly defined problem of deciding the relative importance of the four columns.

Fourthly, the analysis records the data on which the decision was based, the way in which it was reached, and the objectives of the decision; consequently it shows clearly the need for any change of decision arising from future alterations in the data or the objectives.

A practical illustration of this last advantage can be seen if we consider the choice of the project to follow number 10 which we have already selected. The situation has now changed insofar as the completion of project 10 will have dealt with both the "evaluation of equipment" and the "release of a spinning machine". Consequently the choice for the second project will be that with the highest total score for Quality and New Markets, i.e. projects 1 or 7. These are not, it will be observed, the projects with the second highest rating in the first analysis.

The General Problem

This example has illustrated what can be gained by analysing the situation when we have to make a decision between a homogeneous set of alternative courses of action. However, the situation in this example did not contain some of the commoner complications. Thus we neglected any differences in effort or expenditure between the competing projects. Again, we did not have to place a financial value upon the outcome of each project because the situation was such as to permit the clear-cut directives on the order of importance of the various columns.

We shall now examine the general problem of assessing priorities for technical effort where the competitors differ more widely, are not homogeneous, and so do not permit the simplifications present in our example above.

In general the priority of each rival project is dependent upon:

1. The definition of the objective.
2. The value of the outcome if successful.
3. The expectation of success.
4. The cost of the resources to be expended.

The next part of the paper is largely devoted to examining the practical ways in which we can determine these factors.

Definition of the Objective

When we choose a course of action A, rather than B, C or D, it is because we expect* a better outcome from it than from any of the alternative actions. The definition of "better" in the particular situation, i.e. the statement of the objective, provides the yardstick on which we must compare the priorities of the different alternatives.

Many decisions in real life have objectives which are not just those of maximum expectation of gain. For example, you are invited to choose between (i) a shilling and (ii) a 50–50 chance of half-a-crown depending on whether it falls heads or tails. You may reasonably be expected to choose the second alternative with its actuarial value of 1s. 3d.

But in a slightly different situation, where you have no money and need a shilling for your fare home, you might well take the first choice and sacrifice the actuarial prospect of an extra 3d. as the cost of the certainty of having your fare.

It is, therefore, an important first step in analysing a decision-making situation to discover the precise objectives and their relative importances.

Assessment of Values

The task of placing a financial value upon a project is least difficult when its objectives can be defined in terms of previous experience. If we are contemplating doubling the output of an established product from a factory we need to know the expected profit, i.e. our new costs and our new selling prices. The manufacturing costs at the new level will often be quite accurately predictable from the existing records of those at the old. The new selling price of the product will be predictable, but with rather less accuracy because of uncertainty about the response of both the customer and of competitors to the new scale of selling. Nevertheless, we can expect to make a sufficiently accurate estimate of the overall profitability for practical purposes.

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* It has been frequently argued that probabilities in this frequency-ratio sense are quite inapplicable to the making of a decision; the decision-taker being concerned with the outcome on the particular occasion and not with some infinite series of hypothetical decisions. If we accept this view we are driven towards non-Bayesian statistical decision functions of the kind developed by Abraham Wald which ignore any beliefs we may have that one outcome is more likely than another.

But the consideration of expected outcomes finds support from Professor Shackle (notably in his book "Expectation in Economics") who postulates "anticipated surprise, or disbelief at a particular outcome" as an (inverse) measure of the expectation of the event.[7, 8]

VOL.
5
1954

It is more difficult to place a value upon the development of a completely new product. We know less about its ultimate selling price because we frequently are looking further into the future and we cannot extrapolate from existing figures. A useful measure of the effective value of a new development of this kind can be obtained by predicting the income from it and subtracting the best income which could be obtained by using the same capital to expand the existing business. For example, *profit anticipated over, say, the first* 10 *years* minus *the cost of research and development* minus *profit from investing the total expenditure at the company's marginal rate of profit*. The period of profitable activity must be estimated in each particular situation and will be determined by such considerations as the expiry of patents or the fruition of further research which will make the development obsolete.

A still more hazardous task is that of estimating the value of technical effort expended, for example, on technical service to customers, or on fundamental research. Perhaps it is better to consider the probable losses which would result from failing to carry out the technical service or fundamental research. But, on either view, the errors are so great as to make estimates almost valueless for any individual project. Over a long period, too little research leaves the company falling behind the leaders of its industry in technical improvements, while too much research leaves the company with numerous potentially profitable innovations which cannot be exploited for lack of capital or other resources. But these criteria are vague and scarcely give a precise guide for the current budget.

We are left with the situation when that the errors of estimating in advance the profitability of individual research projects are excessive while the criteria for deciding the right amount of research as a whole are vague. Does this mean that we must abandon any attempt at estimating the value of, say, fundamental research? Clearly it cannot mean this because, difficult though the task appears, the yield from research is tacitly estimated or assumed when each and every decision to allocate money to a project is made in industry. In other words a company does not proceed with a piece of research unless it tacitly assumes that the expectation of profit from it will exceed the yield from investing the same money in the company's existing manufacturing activities.

In practice, the inaccuracy of the estimates for individual projects suggests that we should apply our evaluation to whole fields of activity such as technical service or long-term research rather than the individual project. When we do this it seems that

it is preferable to examine the ratio of value to cost (i.e. the profitability, or reward—effort ratio) rather than the value alone.

Relationship between Effort and Reward

Such a reward-effort ratio may, however, vary as we change the proportion of a company's resources which the effort absorbs. For instance, the Standard Oil Development Company have reported a 15-fold return on the money expended (say $30M annually) on Research and Development. (Ref. 13, p. 294.) Now this ratio of 15 would not necessarily apply at an annual expenditure of $100M instead of $30M owing, for example, to insufficient resources for exploiting the technical advances. Likewise, there is a change in the chances of success resulting from additional effort put into an individual technical project depending on the total amount of effort already put in.

We shall, therefore, examine generally how the marginal expectation of profit (i.e. the expectation from an additional unit of effort) varies with the amount of effort already expended and consider how this relationship varies from one sort of activity or project to another and for what reasons.

Decreasing Returns

One knows from experience that some projects are of the kind where the expectation of success falls off progressively if one has already expended a great deal of effort unsuccessfully. If one eventually abandons the project one has tacitly judged that the increase in expectation of success for a marginal increase in the effort no longer justifies further work.

The situation can be conveniently represented by a reward–effort curve of the type shown:—

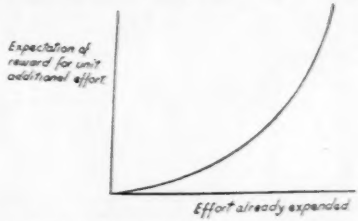
This first type of curve applies to, for example, empirical "trouble-shooting" in a factory, consisting of trying to put things right without finding the cause. Here, the most probable direct remedies for the trouble are examined first; if these first guesses do not reveal the source of the trouble the expectation of success from further empirical effort diminishes.

VOL.
5
1954

The same line of thought can be applied to a whole class of activity, as opposed to an individual project. For example, a research team entering the empirical search for new "wonder drugs" from moulds at the present time will have a low expectation of success compared with ten years ago. The position on the effort-scale is fixed by the total effort already put into this field by all investigators; the marginal expectation of success is that of the additional investigator in the field, who must work on the less promising moulds which remain unexploited.

Increasing Returns

The second curve is characteristic of a factory investigation in which the field of search is systematically and progressively narrowed, and where it is known that the objective does lie within the field of search.



This pattern is also characteristic of whole classes of activities such as research and development where "know-how" is built up and progressively increases the marginal profitability of additional effort. In some fields the curve may be quite flat for a long time, i.e. a great deal of money must be spent on acquiring special equipment, techniques and background data. For example, some millions of pounds can easily be spent on the research and development necessary for deciding whether or not to go into production of a new synthetic fibre at all.

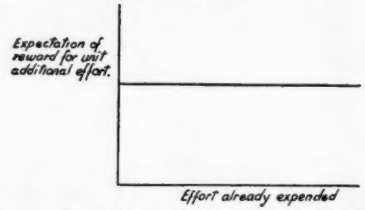
To illustrate very simply the difference between this method of enquiry and the previous empirical one, let us examine a motor car which has broken down. By the empirical method one looks immediately for the most likely direct cause, such as a blocked jet or a broken wire. By the second, the field-narrowing method, one gradually limits the field to, say, the electrical system or the petrol supply, and then to, say, the carburettor, before proceeding to isolate the precise fault. The first method is essentially hit and miss; the second painstaking. In the first one may find the answer straight away and with relatively little effort—but also one may not find it at all; by the second, one will not find the answer straight away, but one will, without doubt, find it in the end.

The decision on when to switch from empirical and unsystematic trials of possible cures to a field-narrowing search for the cause can be considered as a comparison between curves of the two forms. Neither of these forms of investigation is inherently more profitable than the other; the essential requirement is that each should be used when and where it is appropriate.

One of the greatest virtues of considering the reward-effort relationships for projects may be to ensure a recognition of those changes of circumstances which demand a revision of priorities and a changed deployment of our effort.

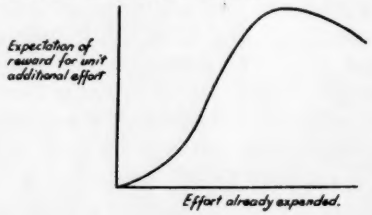
Linear Returns

The third class, the linear relationship, applies perhaps to long-term undirected research where the field of investigation is effectively unlimited and previous success or failure has little influence.



We have discussed so far, only the form of the initial parts of the return-effort curves. These are adequate for projects whose objectives can be completely defined and are known to be attainable such as where the task is to restore a previously existing situation or to achieve uniformity, because here the maximum reward has a clearly defined upper limit, i.e. the solution of the problem.

For all classes of effort where the ultimate objective is not clearly limited, the curves eventually flatten out. This happens because the reward becomes increasingly limited by some new variable besides the effort expended. For example, advertising may stimulate an increased demand but production may not be able to expand fast enough to meet it; again, research may reveal a profitable new manufacture, but finance or labour may not be available for putting it into production.



VOL.
5
1954

It is of some academic interest to classify technical activities into these various forms—but has it any practical use?

The short-term advantage is in an attitude of mind; we are compelled to recognize and obtain information on all the factors relevant to the particular strategic situation. Thus a clear recognition that we are expending our effort in such a manner which will give little quick reward but which promises ultimate success emphasizes that we should expend some effort to make quite sure that ultimate success is as certain as we think it is—more formally, to ensure that the ultimate objective really is within our field of search. Conversely, by ascertaining that our competitor really has succeeded in solving a difficult technical problem, we can justify our own research being more exhaustive than if we were still in doubt as to whether or not a solution really existed.

Again, if we are empirically trying out different possible cures for a factory problem whose cause is unknown, we must constantly ask whether we should yet "cut our losses" and change to a systematic investigation and try to find the cause. Or, if we are about to put into production a process of apparently satisfactory profit ratio, we must ask whether the profit will not be limited by some new factor such as increased raw material costs or shortage of labour, which arises at the particular scale we are contemplating.

Now the advantages we have just looked at are essentially tactical and short-term, but the advantages in the long-term of this questioning attitude of mind are also considerable.

It is clear that we could deploy our technical effort better if we knew precisely what the profitability would be for each kind of activity—for, say, fundamental research or for development, as compared with spending the money on expanding the existing business or by investing it in securities of known yield. We saw that it was often difficult to arrive at useful estimates of the value of individual projects in, say, fundamental research but that a figure could be obtained if we took the average over a number of such projects. We have seen in addition, that the profitability is likely to change considerably in response to many factors such as the type of project and the proportion of a company's resources which are engaged. Therefore, a study of these reward-effort curves is the simplest way in which we can obtain quantitative information of practical value on the relative profitability of the competing projects.

The best decisions on the proper deployment of technical effort are those based on sound experience so that it is sometimes argued that this analytical approach is unnecessary. What must be remembered, however, is that all the subjective judgments *implicitly* assume precise figures for the profitability of the different projects; the judgments would inevitably be improved if the actual figures were available. Thus a company may decide that next year it will spend, say, £200,000 on developing new processes, £100,000 on fundamental research and £50,000 on technical service out of a gross profit of £3,500,000. If the decision is a sound one it is implicitly assumed that the (marginal) reward-effort ratio for each activity is the same at the particular levels of annual expenditure; in other words, the overall expectation of profit to the company would be reduced by making any other division of the available resources such as increasing the allocation of money to each activity by £50,000 or by transferring £50,000 from development to fundamental research. If the marginal gross profitability of the capital invested in manufacturing is 30 per cent, then it is implied that this is the marginal profitability of research, development and technical service at these particular levels of expenditure. If this is not true, then a better deployment could have been made.

We must conclude, therefore, that the provision of more quantitative information would facilitate the sort of decisions which are regularly made in most companies.

Action Suggested

A very limited amount of information has been published by certain companies about their ratio of reward to effort on research and development. For example, we quoted earlier the Standard Oil Development Company (a subsidiary of Standard Oil, New Jersey) which has published figures based on 10 years' experience on a large scale (2,000 graduates) showing a 15-fold return on the money expended on Research and Development; the returns are made from savings in royalties (25 per cent), profits from improvements (65 per cent) and profits from new processes (10 per cent). (Ref. 13, p. 294.) Other companies have quoted very different returns. DuPont reported a 4:1 return from $30M per year spent on Research and Development whereas American Cyanamid had only a 1½-fold return on $10M annual expenditure (Ref. 13, p. 294). What we do not know is whether the 10-fold variation in these returns is due to differences between the industries, in the methods of calculation, in the percentage of turnover spent on Research and Development or to other causes.

VOL.
5
1954

The most practical step may be for individual companies to record retrospectively not just the costs of technical effort but also the estimated rewards gained, and losses avoided, for every project. The large errors in estimating some of the individual returns will be balanced out by accumulation of information on a large enough scale.

The reward-effort ratio may depend upon many factors, such as the size of the technical team and diversity of skills within it, the time taken to complete the project, the size and structure of the company, the nature of the industry, the actual and maximum possible rates of expansion of the company, and so on. What is needed is the accumulation of such information within organizations and its subsequent publication.

Within only a few years, enough information could be accumulated within a large organization to show the dependence of the profitability of technical effort on these different factors. The publication of such figures by British industry generally would greatly increase and accelerate their usefulness. Where a company feels that publication of, for example, the amount of its research expenditure would help its competitors, the publication of reward-effort ratios alone would be useful.

To conclude the paper, I would like to summarize the main arguments:—

Firstly, a semi-quantitative analysis of the relevant facts, such as we made in the example in the first part of this paper, can simplify the task of deciding priorities for technical effort.

Secondly, the difficulty of placing a value upon the expected results of technical effort is the main obstacle to making a precise quantitative analysis for arriving at decisions on priorities, although such values are, in fact, implicitly assumed in all practical budgeting.

Thirdly, systematic accumulation of data on the reward-effort ratio for completed projects would rapidly provide an increasingly precise picture of how the profitability of technical effort depends upon factors such as the nature of the industry and of the technical effort, the size of the company or the diversity of skills in the technical team.

Lastly, this information on the circumstances governing the profitability of technical effort would lead to a better use of existing technical resources, both in men and equipment.

VOL
5
195

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CRIMINOLOGY AND PREDICTION

by

L. T. WILKINS

OVER many centuries many forms of punishment and treatment have been tried with the joint objectives of reforming offenders and deterring others. It remains a fact, however, that after any form of treatment a proportion of offenders are found before the Courts again. Since a large proportion of the total amount of crime committed, particularly serious crime, is committed by those who have committed crime before, the problem of recidivism is a serious one. Despite this fact, the changes which have been made in the penal system in this and other countries have been made on humanitarian grounds and not because any one form of treatment was known to be the more effective than another. The Courts doubtless make every endeavour to select the treatment which they believe to be most suitable for the offender, but there is no evidence to assist them in their judgments.

In the United States where the indeterminate sentence is usual and parole is determined by parole boards, some attempt has been

VOL.
5
1954

made to provide data of guidance to the boards in their determination of suitability for parole.[1] In general, their attempts have been inspired by the actuarial method and no attempt to use statistical decision theory has so far been made. In 1951 the Prison Commissioners asked Dr. Hermann Mannheim the well-known criminologist and myself to look into the methods of criminological prediction and to attempt to build a system for predicting the likelihood of the success or failure of Borstal boys. In the course of this project some adaptation of statistical theory was made which might have a wider interest. The main purpose of the study was to predict the likelihood of failure *after* Borstal training using only evidence available *before* committal.

Since the only clear simple and objective criterion of success or failure after training is whether, after release, the lad either is or is not convicted of a further indictable offence, our main model is afforded by the discriminant function. There are, however, a number of technical difficulties in applying this model to the type of data available from the official case histories of Borstal boys.

First Prediction

The experience upon which the tables* were constructed was derived by sampling one in every three lads committed to Borstal from mid-1946 to mid-1947. It proved possible to obtain information about 60 or so items which have been considered by previous workers in criminology to be associated with recidivism. Fortunately, for the statistician, if an offender is likely to be convicted again he is likely to be so convicted within a short time after release. It was also shown from the Borstal research that the sooner a lad was reconvicted, the more serious was his offence likely to be. By 1952, therefore, it was possible to divide our sample into two parts—those who had failed within three years and those who had not. Other criteria of degree of failure were used as supplementary evidence and were (i) number of crimes committed after release, and (ii) period crime-free after release. The supplementary criteria enabled multiple regression methods to be used and the results compared with that derived from the main discriminant analysis. The advantage of this will be seen when we have stated the methods in more detail and quoted some of the results.

It was considered to be uneconomic to enter into the information matrix all the 60 or so items of information which were collected

* Although criminologists use the term "prediction tables", the tables might be more correctly termed "experience tables"; their use for prediction purposes raises some problems which we have also made some attempt to consider.

and only those items which reached the significance level $p = 0{\cdot}95$ for the zero order correlation were so used. The presence of a number of dichotomies (e.g. had or had not been on probation, had or had not come from a "broken home" etc.) necessitated some modification of the multiple regression and discriminant model. Similar difficulties were also caused by unscaleable categories such as occupation. This latter difficulty was dealt with by cutting the category classification at the median or as near to the median as possible; thus, for example, occupations classified as unskilled or labouring formed the lower half of our dichotomy whilst all other occupations were in the upper half. All category classifications were similarly reduced to two parts.

The first stage of the solution was derived by taking only the dichotomies into a matrix, leaving all variables out at this stage. The best estimate of the criterion was then derived using only the dichotomies. Thus we had a "score" based on the weighted sum of all attributes. This "score" was calculated for each case and punched on to each individual's card. This "score"—a best estimate of likelihood of failure using attribute data only—formed a dummy variable. This dummy variable was then taken into other matrices together with the real variables and the solutions then proceeded in the usual way. The solution then derived gave weight to the dummy and a weight to each real variable. It remained to distribute the weight of the dummy over its constituent parts so that we could end with a simple sum. The empirical distribution was used to achieve this. Using this method we have a best linear estimator of the probability of each individual to fail. In Table 1 below the total score has been used to divide the sample into five risk groups, the size of each group being determined by dividing the sample where the estimates of probability cut the odds of, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{7}{8}$. In particular the centre group may be regarded as a residual category since the correctness of a success or fail prognosis cannot be made with more certainty than $\frac{1}{3}$ for any case. These cases were regarded at this stage as unpredicted.

It is worth noting not only the items which appear at the top of the table, but some which do not appear. Those which obtained no weight include such items as, "broken homes", overcrowding, religion and many others known to distinguish delinquents from non-delinquents. In addition we may note that no subjective judgment proved to be of value in the prediction equation although quite a number were examined. There is some evidence that people are inclined to believe that items which discriminate delinquents from others offer also a basis for assessment of delinquents. It was also

VOL.
5
1954

found that when items were included in subjective judgments they were often given incorrect weights. Although Governors' and Housemasters' assessments are not available before the lad enters Borstal and were not, therefore, used in the main prediction equation, they afford an illustration of this latter point. Both assessments were

TABLE I

ITEMS USED AND RESULTS OF THE FIRST-STAGE CLASSIFICATION INTO GROUP ACCORDING TO RISK OF FAILURE

Items used	Maximum possible score (weight)
Previous convictions	45·0
Longest period in any one job	11·7
Living in industrial area (Industrialization Index[2])	8·0
Living with parent(s) or not	7·5

Results

Risk Category	Raw score (range)	Results of Borstal training: Success	Results of Borstal training: Failure	Number of cases		Success rate (per cent)	
A	0– 9·9	47	7	54	129	87	75
B	10–14·9	48	24	75		67	
X	15–23·9	96	65	161		59	
C	24–39·9	28	54	82	98	34	30
D	40 or more	2	14	16		13	
All cases	.. ..	221	164	385		57	

strongly associated with the lad's behaviour in Borstal, and, although behaviour during training is of some predictive value, the subjective judgments gave this so much weight that the partialling out of behaviour from the assessments improved their predictive value.

Validation Study

In order to demonstrate that this type of prediction table was likely to prove useful for other Borstal intakes than that upon which it was based, a further sample of Borstal boys was taken from those entering Borstal eighteen months after our first sample period ended. Because some cases were lost in the first analysis, we ensured on the second occasion that every case was traced. Using the prediction equation we "scored" each lad on information available from his case papers and subsequently checked the result. The agreement between the prediction and the number of successes actually occurring was almost too good.

The Centre Group

No attempt was made at "predicting" the centre group which we called group X in Table I. They stood about equal chances of success or failure. This centre group has worried most criminologists but we made some attempt to predict these "unpredictables". Our first prediction was derived by the solution of equations, and there

seemed no reason why we should not set up another set of equations for the centre group alone. All cases scoring between 15 and 23·9 were accordingly separated and, again selecting items of information on the basis of their zero order correlation being significant, we solved again in the same way as before. Because we needed more information for this second matrix we lost a number of cases and the result is, therefore, only tentative. The procedure resulted in a successful discrimination so that cutting at the optimum point gave us two groups (both within the centre group) where the chance of success of the better half was twice that of the worse. The method thus seems rather promising and some interesting features emerge from this procedure. The items which proved of value in discriminating the better from the worse amongst the centre group are shown overleaf. Not so much the level of the items was prognostic but

TABLE II

COMPARISON OF THE PREDICTED SUCCESS RATE AND THAT ACTUALLY ACHIEVED WITH A FURTHER GROUP OF CASES WHEN CLASSIFIED INTO RISK CATEGORIES

Risk Category with its predicted success rate (per cent)		1948 Intake at a Borstal reception centre Subsequent successes	Subsequent failures	Actual success rate (per cent)
AB	75	90	30	75
CD	30	27	63	30

rather whether there was "conflict" between the levels. For example, *low* intelligence, plus *low* level job, plus *low* level of job of head of household suggested success equally with *high* level job, plus *high* intelligence, plus *high* level occupation of the head of household.

	Weight
Intelligence (Columbian test score)	2
Last occupation of lad	2
Leisure activities	4
Family crime record (father, mother, sibs) ..	1½
Occupation of head of household (usually lad's father)	1

Where the signs were conflicting the chance of success was reduced. It is interesting also to note that the first set of equations (Table I) was weighted almost entirely by past convictions, whilst in the second equation where we had a group of lads who were very similar to each other in terms of past crime record, the items which appeared were in rather a different category. It has been suggested that the second set of items are those which reflect personality differences.

Another hypothesis-provoking finding was obtained by comparing the results of the discriminant solution based only on success or failure as defined, with the solutions derived from multiple

VOL.
5
1954

regression methods using degree of failure or success as the criterion. We could, we found, predict any of the three criteria by the same set of information and by using the same weights. In estimating the probability of failure we were also estimating the degree of failure or the duration of success.

Uses of Prediction

Prediction tables, of course, say nothing about "the causes" of crime, but they enable decisions to be made on a rational basis. When further sets are available for other treatments they will be of more value. The Courts, for example, would not be assisted very much merely to know that if sent to Borstal a certain probability of success was estimated, but when they can compare this estimate with estimates for other treatments they will be able to function with a greater amount of information to guide their decisions.

A sentence to Borstal does not determine how long the lad shall be detained in an institution nor does it specify whether it shall be of the open or closed type. These two features are decided by the administration. We were accordingly able to examine the difference between open and closed treatments and the duration of detention. The success rate for open Borstals in our sample was 58 per cent and that of closed, 36 per cent. We also found, however, that 74 per cent of the (AB) groups and only 39 per cent of the (CD) groups went to open treatment. We should thus expect better results—they had better material. We may, however, using our prediction equations, attempt to partial out the difference expected because of the better material. The results of calculating the within risk group success rates are shown in Table III.

TABLE III

COMPARISON OF SUCCESS RATE FOR OPEN AND CLOSED BORSTALS USING THE PREDICTION TABLES

Risk category	Success rate (per cent) Open Borstals	Closed Borstals	Excess successes in open Borstals
AB	78	67	11
X	61	56	5
CD	38	28	10
All cases	58	36	22

Although the overall difference between the open and closed treatment after making this allowance for the type of lads received is reduced from 22 per cent to 8 per cent the difference remains significant. It is not claimed that this observed difference is due to treatment as such. It is, however, a useful pointer for further research.

The period of detention in Borstal may vary between nine months and three years. On our first examination of the effect of

the period of detention we found, not surprisingly, that the longer a lad was detained, the more likely he was to be a failure. This was due to the worse risks being detained longer on average, perhaps because of the poor conduct within the Borstal, a factor which we have earlier noted as significantly associated with failure. When we examined the results within our risk groups we found that periods of detention had no effect upon the success rate.

In constructing the Borstal prediction tables we have followed the following rules:—

1. Prediction tables must be reproducible by others. This involves using factual data rather than subjective judgments, since such judgments are not reproducible. Moreover, we have no experience of subjective data adding anything to the predictive value of factual data efficiently utilized.

2. The tables must be statistically efficient, making use of only the smallest number of items. In social research, items of information are expected to overlap and inefficient treatment of overlapping items has been a characteristic of all previous criminological prediction work.

3. The final tables must be very simple to operate so that errors of calculation are avoided.

4. Continuous revision is necessary. This is best likely to be achieved by a sequential method. Sequential methods have added attraction in this field due to (*a*) those most likely to fail at any time are likely to fail early after discharge, (*b*) the earlier the failure the worse the failure is likely to be. Thus a sequential validation procedure can not only use information as and when it becomes available, but can also make use of the fact that it becomes available at a certain time.

In conclusion we must acknowledge the work of Wald[3] and Lindley[4] whose emphasis of the idea of decision making in statistical theory and methods provided the inspiration for much of the statistical work in this project.

The Borstal research project from which the material in this paper is drawn will shortly be reported in full in a book in the new series "Studies on the causes and treatment of delinquency—Prediction methods in relation to Borstal Training" by Hermann Mannheim and Leslie T. Wilkins to be published by Her Majesty's Stationery Office.

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VOL.
5
1954

ABSTRACTS AND REVIEWS

How Operational Research Aids Production Control.

STAFFORD BEER. (Saml. Fox and Co. Ltd.)

Business, Jan., 1954; **84** (1), 70–72.

This short article, condensed from a public lecture, points out that commensurable measures of output are difficult to obtain where products vary in size, weight and cost. It therefore suggests that a job productivity index can be used as a measure, that production forecasts can be successfully based on statistically controlled groups of such indices, and that these forecasts can be used to turn static flow models of production into dynamic plans. The components of the productivity index are discussed in terms of mathematical models, in illustration of which an example is quoted from the continuous processing of cold-rolled steel strip.

Operations Research for Management.

CYRIL C. HERRMANN (M.I.T.) and JOHN F. MAGEE (A.D. Little).

Harvard Business Review, July–August, 1953; **31** (4), 100–112.

O.R. is defined as a new concept in management which helps to single out the critical issues which require executive appraisal and analysis and provides factual bases to support and guide executive judgement. Operations are considered as an entity—the combination in total of the equipment used, the morale of the participants, and the physical properties of the output. Operations so conceived are subject to analysis by the processes and methodologies associated with the research work of scientists.

A number of examples are given to show how the scientific method of analysis has been profitably applied to business problems including the distribution of sales effort, warehousing schedules to reduce freight costs, the effectiveness of advertising for different products, the establishment of production time standards for efficiency controls, and the maintenance of balanced stock levels.

Four concepts of fundamental importance to the practice of O.R. are considered; these are (a) the model; (b) the measure of effectiveness; (c) the necessity for decision; (d) experimentation.

The relationship of O.R. to other management services such as statistics, accounting, market research, and engineering are briefly discussed and further sections refer to management problems concerned with establishing O.R. and its limitations and future.

The Film Applied to Work Study.
MARCEL DELFOSSE (Renault, France).
Science and Film, September, 1953; **2** (3), 21–27.

This paper was read at the International Scientific Film Association's Industrial Committee's conference in Paris in May, 1953, with four Renault films (summarized and discussed at the end of the paper) as illustrations.

The author dwells on the advantages of the film in providing a concentrated, permanent and fruitful means of work study and movement analysis.

He emphasizes the importance of correct selection of film speed, and points out that 1,000 images per minute provides 10 images per 0·01 min., a unit often used in rate fixing.

Where team work or flow production is studied, groups of operatives can be filmed together. In such cases 100 images per minute is sufficient, so long as speed is controlled.

Of the four films shown, two compare the performances of five operatives respectively on pre-studding a hinge, and cutting up a bronze bush on a lathe. It is shown that about one-third of the workers filmed had an efficient work method. The third film shows the improved method of pre-studding based on the first film, and the fourth shows the team operation of general body assembly.

The Third Phase.
Tydschrift voor Efficiente Bedrijfsorganisatie, 1954; **9** (3), 65–67. (In Dutch.)

With reference to the recent changeover from Operational Research Club to Operational Research Society, the article describes the ever-increasing scope of operations research. In the cycle of operations consisting of a first phase (planning) and a second phase (execution) operations research may be regarded as the third phase which may provide valuable information for the first phase of the next cycle.

Development and Results of Operations Research.
MANFRED KNAYER.
Zeitschrift die Vereines Deutschen Ingenieure, 15 April, 1954; **96** (11/12), 1–4. (In German.)

The idea of the designer, the foreman's ability to improvise and the feeling the merchant used to have in his finger-tips have decided for ages the success or failure of operations and enterprises. But the good idea did not always come up in time, nor common sense always

VOL.
5
1954

indicate the right direction. Enterprise and risk are closely joined. To keep risk at a minimum, chance must be replaced by planned success. More and more, management uses scientific methods to improve the success of their operations. Methodical and systematic approach is used, not to displace good ideas but to supplement and support them. As the physician uses the oscillograph and the electrocardiogram to be certain about the state of health of a person, so industry, economy and even politics apply new methods known under the name of operational research.

The value of operational research is discussed in relation to productivity and forecasting. Its development and methods are reviewed and some examples of its application to traffic and consumer demand are given.

Social Applications of Operational Research.

R. T. EDDISON (B.I.S.R.A.).

Impact, Summer 1953; **4** (2), 61–82.

The necessity to-day of applying scientific methods to aid decision-making is traced to the increasing complexity and the increasing importance of the decisions that have to be made. The war produced conditions which were ideal for the application of scientific methods to the study of military operations and so operational research came into being.

The application of operational research in industry carries important social implications by ensuring that decisions are made objectively and directed towards improved productivity in the interests of all sections of the industry and of the public. Among social applications reference is made to queuing problems of patients at hospitals, to the evaluation of road improvement schemes and the derivation of regulations governing road usage, to problems of communication and accidents, and to the function of government.

Some Statistical Methods Used in Studies of Steel Plant Operations.

A. P. WOODS (Armco Steel Corporation).

Blast Furnace and Steel Plant, June 1954, **42** (6), 649–652.

This paper describes the use of graphical multiple correlation methods in the analysis of uncontrolled data obtained from actual plant operations. The method is applied to determine the effect on rate of output of an open-hearth steel furnace of variations in the percentage hot metal charged, the percentage carbon at melt and the scrap charging rate.

Studier i Skogsbrukets Transporter (A Study of Timber Transportation).

ULF SUNBERG.

S.D.A. Report; Vol. **48.**

This paper by Professor Sundberg is written in Swedish with a summary in English.

The first part of the paper deals with costs. Two classifications are given: historical costs are those which have been incurred in the past and alternative costs are those which are expected in the future or which would result from choosing alternative methods.

The author deals with alternative costs only and his purpose is to give the timber transportation process an optimum form which corresponds to the production process that gives the lowest cost. In theoretical analysis one or more production factors are varied, the others remaining constant. This will give the optimum combination.

Under such dynamic conditions some costs do not change—constant costs; while others vary—variant costs. Costs can change continuously or they can change by leaps, when the production amounts to a certain quantity.

The logging process is broken down into cutting and transportation and in this latter, two groups of costs are distinguished: transit costs, and terminal costs.

Transportation costs can be connected either directly or indirectly with the logs, thus giving a break-down of total cost of direct and indirect cost of transporting the logs, plus overheads.

The direct and indirect costs are considered in turn.

The second part of the paper deals with the co-ordination of the transport system, and the plan of the discussion is summarized under three points. Firstly, the most economical transport, taking account of variations in the quantity of the wood, in different sections of the road from stump to mill must be found; secondly, it must be decided if any transport should be eliminated and thirdly, the different transports should be co-ordinated.

The principles of elimination are discussed and then the paper considers at length the co-ordination of two transports, the primary and secondary, to move the logs from the felling ground to the mill.

The general discussion is then applied to the co-ordination of different timber transports in Swedish forestry.

Numerous formulæ are developed and used throughout the discussion and in the examples of the application of the general discussion to specific problems.

The final chapter of the paper deals with the optimum standard

VOL.
5
1954

of road for the job. This problem is dealt with in a simplified form because very few data are available for making a thorough analysis, and the results are, to a certain extent, based on hypothesis. However, the author introduces several formulæ which are used to determine the optimum, and supports them wherever possible with time studies and statistics.

Design and Analysis of Industrial Experiments.
Edited by OWEN L. DAVIES.
Oliver and Boyd. Edinburgh, 1954 (*for Imperial Chemical Industries Ltd.*) xiii + 636 *pp.*; 63s.

This book is a sequel to *Statistical Methods in Research and Production* (1947) which dealt mainly with the use of statistical methods for extracting information from pre-existing data. It might be thought that the earlier volume would be of greater direct interest to operational research workers, on the grounds that there is little room for the planned experiment in studying problems during production. The power and efficiency of planned experiments, however, are such that the research worker needs to grasp every opportunity of using them in preference to limiting his work to the analysis of uncontrolled data; and the opportunities for using designed experiments in studies of production are not as infrequent as the faint-hearted may sometimes think, as may be seen from some of the examples quoted in this book.

Design and Analysis of Industrial Experiments is clearly aimed to include the inexperienced among its readers. Starting with the simplest ideas of comparative experiment the principles of experimental design are carefully developed (with the intrusion of a chapter on studying chemical sampling and testing methods) up to Latin Square and Incomplete Randomized Block Designs, followed by four chapters on Factorial Experiments. The book will be found particularly useful for its two last chapters which deal with fractional factorial experiments and the determination of optimum conditions, the latter giving practical applications of the methods developed by Box and Wilson.

The book is excellently produced and the six authors maintain a uniform lucidity of explanation. Theoretical mathematics are avoided (or put in appendices to the chapters) and the explanations are supported by practical exposition with examples worked out in complete detail. There are a glossary of technical terms, 36 pages of useful tables of statistical functions and a good index.

R. T. E.

VOL
5
195

O.R. AND ACCOUNTING

IN the July issue of *The Cost Accountant* the following interesting editorial comment is passed on Stafford Beer's recent paper on "O.R. and Accounting" (*Operat. Res. Quart.*, **5** (1), p. 1).

"As a technique operational research—which has developed greatly in recent years—may be difficult to define, but unquestionably it has great value for industry and its relationship to accounting must be a close one. Without really clarifying this relationship, however, the writer is rather critical of certain accounting and cost accounting conventions (*e.g.*, the practice of averaging overheads over direct labour within a unit), which are not perhaps so widely accepted within the profession as he suggests. Blind acceptance of many normal cost accounting methods certainly can mislead; the important thing is to be aware of their limitations.

"Undoubtedly progressive thought in accounting is fully alive to the weakness of many conventions, which after all have emerged only because they are of practical value in certain given circumstances. That is true of all conventions, their function being to save effort, in the widest possible sense.

"As Mr. Beer points out, accountancy has established its ascendancy in the realm of providing information about the conduct of business to managements. Operational research represents another tool at the disposal of management, to be integrated with the other techniques of ascertaining facts. Of itself it cannot, as Mr. Beer perhaps suggests, replace accounting.

"Many will disagree with some of Mr. Beer's views. He has much to say, for example, about the theoretical bases of standard costs, budgetary control, and various other aspects of cost accounting, but whether one agrees with him or not it is refreshing to have the underlying principles re-examined.

"His paper is a general plea for wider understanding of the use of operational research and about this all accountants will surely agree with him."

The Cost Accountant is the official journal of the Institute of Cost and Works Accountants.

Centre Français de Recherche Opérationelle

The address of the Centre Français is now 115, Boulevard St. Germain, Paris 7e. The address given in our last issue is that of M. Salzmann, the director, and should in any case have read 12, rue de Longchamp (instead of 11, as printed), Paris 16e. We apologize for the error.

VOL.
5
1954

O·R

OPERATIONAL RESEARCH QUARTERLY

VOL. 5 NO. 4 DECEMBER 1954

Contents

Operational Research Society 93
Application of Birth and Death Process to Provision of Spare Machines ——J. TAYLOR and R. R. P. JACKSON 95
Management in the U.S.A. 108
Queueing Systems with Phase Type Service——R. R. P. JACKSON . . 109
Army O.R. in the Far East, 1952/54——K. PENNYCUICK . . . 120
Abstracts and Reviews 130

EDITORIAL NOTES

OPERATIONAL RESEARCH SOCIETY

AT the annual general meeting of the Society on 13 October, 1954, the Committee's report for the year ended 31 July was accepted. The report stated that membership on 31 July was 101, which included 31 new members (out of 34 applications), and allowed for two resignations. After detailing the meetings held during the 1953/54 session and announcing those for the 1954/55 session, the report went on: "During the year the committee have continued to restrict membership of the Society to qualified operational research workers. They see the need however for the Society to cater for people who wish to be associated with operational research, but who would not claim to be experienced workers in this field. Consequently the committee have passed a resolution, for which endorsement will be asked at the Annual General Meeting on 13 October, 1954, to create a grade of Associate of the Operational Research Society. Associates would have the same privileges as full members and would pay the same subscription."

Since the report was accepted, this proposal has been put into effect, and several associate members have been elected.

Dealing with the affairs of this Quarterly, the report stated that we had published four issues during the year under review, "containing a total of 100 pages—more than in any previous year, and comparing with 76 in 1952/53. Circulation continues to grow slowly and steadily, and now numbers 733 (paid for) copies,which go

to 15 countries in all 6 continents. Two hundred and twenty two of the circulation now goes to the U.S.A. where the active and efficient organization of our honorary North American subscription agent, Dr. Peter B. Myers of the Bell Telephone Laboratories has earned the gratitude of the Society.

"With the expansion of the Club into a Society" continued the report, "the Quarterly should now be able to draw on a wider range of industrial experience of operational research projects, and the editors hope that members and readers will put forward articles and reports for consideration, especially those concerned with practical applications."

The committee thanked A. W. Swan, the retiring member, for his work during his period on the committee. They also recorded their thanks to L. T. Wilkins, whose resignation as treasurer they were sorry to receive. E. C. Williams was appointed by the Committee to replace Mr. Wilkins as treasurer.

Dr. H. F. Rance (Wiggins, Teape & Co. Ltd.), proposed by A. W. Swan and seconded by B. H. P. Rivett was the only nomination for the vacant place on the committee, and was formally elected by the meeting, as were the secretary (B. H. P. Rivett), treasurer (E. C. Williams) and auditor (R. G. Stansfield) who submitted themselves for re-election.

After the annual general meeting J. Taylor and R. R. P. Jackson of British Overseas Airways Corporation presented a paper on "An Application of the Birth and Death Process to the Provision of Spare Machines."

THE Society's "open" meeting was held at the Royal Society's Rooms on 20 December. The subject was "The Accuracy and Function of Time Study", and the principal speakers were Professor T. U. Matthew, of Birmingham University, and W. Rodgers, of D.S.I.R. The Chairman was A. W. Swan, of Courtaulds Ltd.

The speakers at the meeting to be held on 18 January, 1955, on "The Handling of Uncontrolled Operational Research Data", will be H. G. Jones, of The Steel Company of Wales, and Dr. G. H. Jowett, of Sheffield University. R. T. Eddison will be in the Chair.

Dr. O. H. Wansbrough-Jones, C.B., O.B.E., chief scientist to the Ministry of Supply and chairman of the Operational Research Society was awarded a K.B.E. in the New Year Honours.

VOL.
5
1954

AN APPLICATION OF THE BIRTH AND DEATH PROCESS TO THE PROVISION OF SPARE MACHINES

by

J. TAYLOR* AND R. R. P. JACKSON†

IN any progressive organization there must be, as the source of its progressiveness, a continuing generation of new ideas and unremitting effort to do things more efficiently and therefore more economically. However, if the success of a business organization lies, as it undoubtedly does in part, in its ability to organize its employees to work together in a harmonious team, then it must follow that some ideas, although good in themselves, may not necessarily be acceptable to the corporate body. This is not to say that such ideas should not be aired but rather to lead up to the usual statement that the authors of this paper are not necessarily expressing the official views of B.O.A.C. Nevertheless, they have to recognize the opportunities and encouragement given to such—perhaps academic might be the word at this stage—work as they have been able to do.

Before describing some of the technicalities of our work on the application of queue theory to the provisioning of spare engines, it will be advisable to put the work into its context within B.O.A.C.

While it is true to say that safety must condition all the thinking of aircraft maintenance engineers, one of their main day-to-day problems is how to plan the work in the hangar so that unscheduled removals of unserviceable items cause as little disturbance to routine operations as possible. Unfortunately, failures are nasty creatures, whichever way they are looked at, and although some allowance for their incidence can be made in routine planning of work, they tend to be very will o' the wisp in their behaviour. In scientific or statistical circles this type of behaviour is known as random, and randomness is the bane of the "practical" engineer who likes to have things working according to plan.

Now except for trying to improve the reliability of an item by progressive modifications, different overhaul techniques and standards, nothing can be done about this randomness. But the disruptive

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* Senior Investigations Officer, and † member of staff of Analysis Branch, British Overseas Airways Corporation. This paper was given to the O.R. Society in London on 13 October, 1954.

effects upon the aircrafts' availability for flying can be minimized by having a sufficiency of spare serviceable units with which to make immediate replacement of the failed item. The important question is of course "how many?" On the one hand, the accountants want to keep the amount of capital tied up in stocks of spares down to as low a level as possible; the engineers, since they have to bear the brunt and parry the brickbats, want to leave nothing to chance and to carry as large stocks as possible. And who shall blame either of them?

It seemed to us in Analysis Branch that here was a fruitful field of investigation to see whether something could be done to reconcile these diverse points of view. The object was to provide a system of planning spares requirements which, while mathematically sound and competent to take the randomness of arisings as well as the known variation of repair or overhaul times into account, should not at the same time call for the employment of senior wranglers as spares planning assistants—even if they could be had for that job, which is open to doubt. It seemed to us, too, that the taking into account of the effects of variations in time for overhaul might cast some interesting sidelights on the importance of this factor in relation to factory organization. As will be seen later, we were not disappointed.

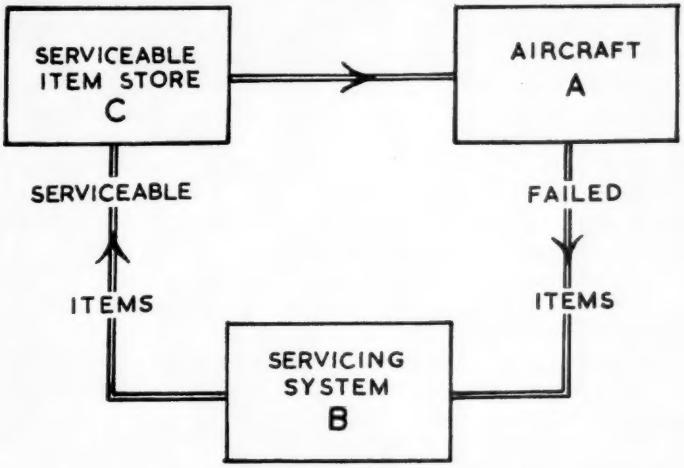
To return, however, to the problem of balancing the claims of the accountants against the natural desires of the engineers. The engineer, of course, wants to make certain that when a serviceable spare is required, one is available in the stores and delay in returning the aircraft to service is avoided. But in the context of random failures nothing is more certain than uncertainty. Accordingly, it seemed appropriate to take as our new planning criterion the number of spares necessary to provide a given standard of operational efficiency, this operational efficiency being defined as merely the percentage chance of having a serviceable spare in stock when one is required. This became known as the "Old Mother Hubbard" problem.

The system, or mathematical model, to be considered is, therefore, a closed circuit illustrated on the following page.

In the course of their routine flying on usually a standard pattern of scheduled services, the aircraft can be expected to generate unserviceable items which accordingly require servicing attention, this taking varying lengths of time to accomplish. As these items for service, arising to all intents and purposes at random, are removed from the aircraft, replacements are fitted from the store. Thus, if n spares in all are available, as time passes n items will be

VOL.
5
1954

distributed round the circuit in varying proportions. Sometimes, if a spell of below average failures has been experienced, most of them will be in the serviceable store waiting to be used. At other times, following a spate of failures, possibly coupled with a run of repairs taking longer than average, the spares will be accumulated in the servicing system itself, either receiving or queueing for attention. But so long as not more than n are unserviceable, even though the maintenance engineer may get a bit touchy, the aircraft can continue



to operate. The chance of the number of items in the circuit not exceeding n is therefore the operational efficiency mentioned earlier.

We proceed now to discuss in some detail the method of attack and to touch upon a few interesting results arising from the solution.

As has been suggested above, a comparison between the number of spare machines available and the corresponding probability of an operational emergency, i.e. being unable to operate the specified number of machines, provides a useful criterion when attempting to solve this particular problem. On the basis of certain assumptions, this paper suggests a model to which the Birth and Death Process equations are applicable and obtains the relevant probabilities which are interpreted in a series of practical examples. Although the systems in which the considered situation occurs are vastly divergent in practice, e.g. a set of machines used for a continuous manufacturing process and, say, a fleet of aircraft maintaining a steady rate of flying (here, of course, machines = aircraft engines), the model can be used in most cases with only minor modifications.

The Theoretical Model

Suppose that N machines are continuously operating in zone A in the diagram and, once a machine becomes unserviceable, it is:

(i) instantaneously replaced by a spare from the store C (provided, of course, that C is not empty), and

(ii) instantaneously transferred into the servicing circuit B.

The N machines in A are assumed to become unserviceable at random but at a constant average rate f per unit time. The servicing time for a machine is also taken as random with an average of k units, the time in any particular instance depending upon whether a major overhaul or a relatively minor repair is necessary; further, it is supposed possible to service r machines concurrently.

If the total number of spare machines available be n, then at any time t when the N machines are operating, there are n machines distributed between B and C. Provided that not more than n machines are held in the servicing circuit B, N machines can continue to operate. However, once $n+1$ machines are in B, then it is no longer possible to operate the specified number of machines. At this juncture, two possibilities exist, either:

(i) the machines cease operating until a serviceable replacement becomes available from B to restore the operational number to N, or

(ii) a number $< N$ of machines continues to operate.

We are here limiting our considerations to case (i) only, in accordance with our basic assumption, although it is clear that case (ii) will often represent practical situations and, as such, deserves separate treatment. For present purposes, therefore, the probability of an operational emergency is the same as the probability that there are $(n+1)$ machines in the servicing circuit B and therefore only $N-1$ machines available for operations.

On this basis, let the probability of having i unserviceable machines in B at any time t be $P_i(t)$. It may then be shown that the whole system is characterized by the following set of $(n+2)$ differential equations:

$$k\frac{\partial}{\partial t}P'_o(t) = -fkP_o(t) + P_1(t) \qquad i = o$$

$$k\frac{\partial}{\partial t}P'_i(t) = fkP_{i-1}(t) - (fk+i)P_i(t) + (i+1)P_{i+1}(t) \qquad o < i < r$$

$$k\frac{\partial}{\partial t}P'_i(t) = fkP_{i-1}(t) - (fk+r)P_i(t) + rP_{i+1}(t) \qquad r \leqslant i < n+1$$

$$k\frac{\partial}{\partial t}P'_{n+1}(t) = fkP_n(t) - rP_{n+1}(t) \qquad i = n+1$$

These equations are similar to those given by Feller[1] and, for given initial conditions, may be solved for any t in the normal manner. In practice, however, only the steady state solutions are of interest, and these are independent of the initial conditions.

The steady state solutions, or the limiting distribution of $P_i(t)$, are given by:

$$P_i = \frac{(fk)^i}{i!} P_o \qquad \text{for } i \leqslant r$$

$$P_i = \left(\frac{fk}{r}\right)^{i-r} \frac{(fk)^r}{r!} P_o \qquad \text{for } r \leqslant i \leqslant n-1$$

Whence, since $\sum_{i=o}^{n+1} P_i = 1$,

$$P_o = \frac{e^{-fk}}{1 - P(fk, r+1) + {}_{fk}P_r \left[\dfrac{1 - \left(\dfrac{fk}{r}\right)^{n+1-r}}{1 - \dfrac{fk}{r}} \right] \dfrac{fk}{r}}$$

where $P(fk, r+1) = e^{-fk} \sum_{i=r+1}^{\infty} \left(\frac{fk}{i!}\right)^i$

$${}_{fk}P_r = e^{-fk} \left(\frac{fk}{r!}\right)^r$$

} Obtainable from tabulated values of the Poisson distribution.

Although interest is naturally centred upon P_{n+1}—the chance of the system breaking down, or perhaps more conveniently, upon the operational efficiency of the system, defined simply as $E = 100(1 - P_{n+1})\%$—this aspect cannot be considered in isolation from other factors underlying the other part of the economic picture. Thus, for any given unserviceability rate f, the effects (and costs) of varying the servicing parameters k and r must be examined in relation to the associated numbers of spare machines necessary for an acceptable operational efficiency. This examination is achieved by considering the effects of the servicing parameters on such items of interest as:

(i) Average number of machines unserviceable

$$= \sum_{i=o}^{n+1} iP_i = U$$

(ii) Average number of machines being serviced

$$= \sum_{i=o}^{r} iP_i + r \sum_{r+1}^{n+1} P_i = S$$

(iii) Average number of machines waiting for service

$$= \sum_{i=r+1}^{n+1} (i-r) P_i = W$$

(iv) Service utilization factor

$$= \frac{100S}{r} = F\%$$

Application

The special case $\frac{r}{k} = f$

The case of $\frac{r}{k} = f$ is that in which the potential servicing rate $\frac{r}{k}$ is equal to the average rate of unserviceable arisings f. For the problem when there is an infinite supply of "potential queuers", i.e. serviceable spare machines, the solution is classic and illustrates the development of an infinite queue of machines waiting for service. In the present example there is a finite number of "queuers", at most $n+1-r$ (unserviceable spare machines), the steady state solutions for this case being given by:

$$P_i = \frac{(fk)^i}{(i\,!)} P_o = \frac{r^i}{i!} P_o \quad \text{for } 0 \leqslant i \leqslant r$$

$$P_i = \left(\frac{fk}{r}\right)^{i-r} \frac{(fk)^r}{r\,!} P_o = P_r \quad \text{for } r \leqslant i \leqslant n+1$$

where $$P_o = \frac{e^{-fk}}{1 - P(fk, r+1) + {}_{fk}P_r(n-r+1)} \quad \text{for } n \geqslant r.$$

It follows that:

$$P_o < P_1 < P_2 < \ldots < P_{r-1} = P_r = P_{r+1} = \ldots = P_{n+1}$$

so that the probability distribution of the number of unserviceable machines is a monotonic increasing sequence.

VOL.
5
1954

Now $P_r = \frac{r^r}{r!} P_o$

$$= \frac{r^r}{r!} \frac{1}{1+r+\frac{r^2}{2!}+\ldots+\frac{r^r}{r!}+(n-r+1)\frac{r^r}{r!}}$$

$$= \frac{A}{B+A(n-r+1)} \quad \text{where } A = \frac{r}{r!} \; B = \sum_{i=o}^{r} \frac{r^i}{i!}$$

$$< \frac{1}{n-r+2} \quad \text{since } B > A$$

Also, $P_r > \frac{A}{B}\frac{1}{n-r+1} = \frac{C}{n-r+1}$ where $C = \frac{A}{B} < 1$

Thus $$\frac{C}{n-r+1} < P_r < \frac{1}{n-r+2}$$

It follows that the rate of decrease of P_r is $0\left(\frac{1}{n^2}\right)$ so that increasing numbers n of spare machines rapidly cease to have material effect in reducing the probability of an operational emergency, P_{n+1}.

Further, examining the average number of machines waiting for service, we have:

$$W_n = \sum_{r+1}^{n+1} (i-r) P_i$$

where $W_n = W$ with n spare machines in the system.

$$= P_r[1+2+3\ldots(n+1-r)]$$

$$= P_r \frac{(n-r+1)(n-r+2)}{2}$$

so that $$\frac{C(n-r+2)}{2} < W_n < \frac{(n-r+1)}{2}$$

Thus, with increasing numbers of spare machines, the number waiting for service also increases at a steady rate y, where $\frac{C}{2} < y < \frac{1}{2}$. Since, as was seen above, little is gained in operational efficiency beyond a certain limit, the system is also likely to be uneconomic, and the only practical alternative, i.e. $f < \frac{r}{k}$, is now investigated.

The case of $f < \frac{r}{k}$

The case of $f < \frac{r}{k}$ is representative of the situation where the potential rate of output $\left(\frac{r}{k}\right)$ of the servicing system is greater than the average rate of arisings (f). Writing, for convenience, $x = \frac{fk}{r} < 1$ the steady state solution for $i \geqslant P$ becomes:

$$P_i = P_o \frac{(xr)^r}{r!} x^{i-r}$$

$$= \left[\sum_{i=o}^{r} \frac{(xr)^i}{i!} + \frac{(xr)^r}{r!} \left(\frac{1-x^{n-r+1}}{1-x} \right) x \right]^{-1} \frac{(xr)^r}{r!} x^{i-r} \qquad (1)$$

$$< \left[1 + x \left(\frac{1-x^{n-r+1}}{1-x} \right) \right]^{-1} x^{i-r}$$

$$= \frac{1-x}{1-x^{n-r+2}} x^{i-r} \qquad (2)$$

Hence, from (1), P_n is a decreasing function of n and i. $(i \geqslant r)$

Further, $W_n = \sum_{i=r+1}^{n+1} (i-r) P_i$

$$< \frac{1-x}{(1-x^{n-r+2})} \sum_{i=r+1}^{n+1} (i-r)\, x^{i-r} \qquad \text{from (2)}$$

but $$\sum_{i=r+1}^{n+1} (i-r)\, x^{i-r} = \frac{x(1-x^{n-r+1})}{(1-x)^2} - \frac{(n-r+1)}{1-x} x^{n-r+2}$$

$$< \frac{x(1-x^{n-r+1})}{(1-x)^2}$$

$$\therefore W_n < \frac{1-x}{(1-x^{n-r+2})} \cdot \frac{x(1-x^{n-r+1})}{(1-x)^2}$$

$$< \frac{x}{1-x}$$

The average number of machines awaiting service has, therefore, an upper bound independent of n and accordingly increasing the number of spare machines does not, in contrast to the previous

VOL.
5
1954

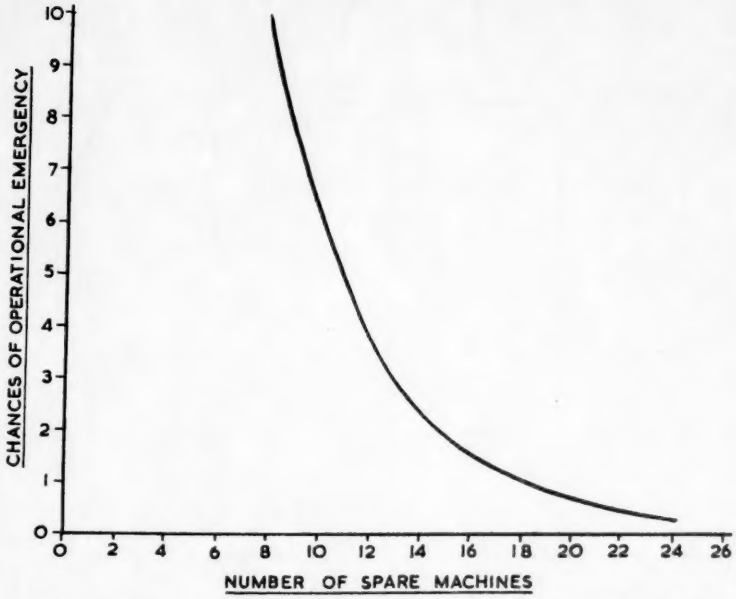


FIGURE 1. *Chance of operational emergency in relation to number of spare machines.*

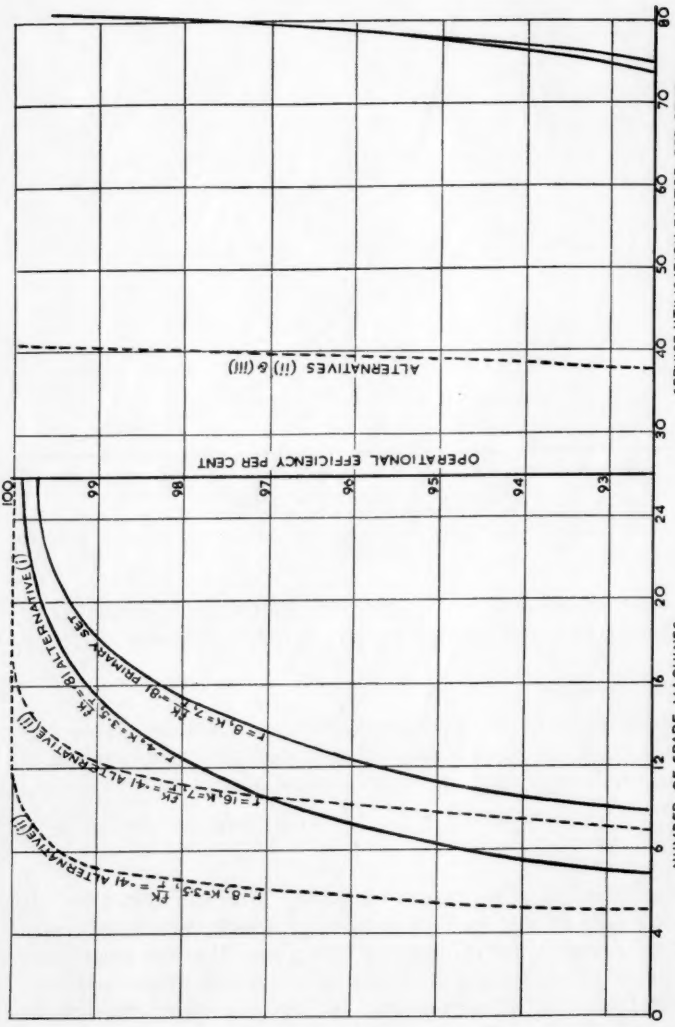
case, increase the number of machines waiting for service. Their effect is directed only towards increasing the operational efficiency.

A specific example

The chances of an operational emergency (i.e. the chances of no spare machines being available when needed) corresponding to various numbers of spare machines is illustrated in figure 1 for the case $f = 0{\cdot}93$, $k = 7$ and $r = 8$, i.e. $\frac{fk}{r} = 0{\cdot}81$, the unit of time being taken as a week.

The variation of operational efficiency with different numbers of spare machines may be seen in figure 2, which also shows corresponding variations for different servicing conditions detailed later. It will be seen that the Law of Diminishing Returns applies and that, in general, it may be uneconomic to seek for more than about 99 per cent efficiency.

In conjunction with the variations in operational efficiency, figure 3 illustrates the utilization factor of the servicing system. In



FIGURES 2, 3. *Inter-relationship between number of spare machines, operational efficiency and service utilization.*

VOL.
5
1954

all practical cases, this cannot reach 100 per cent, but nevertheless the levels below 100 per cent can be used as a quantitative measure of the average level of employment of the servicing facilities; further, although servicing resources may be unemployed on the occasions when there are less than r machines unserviceable, machines will be waiting for service when more than r are unserviceable. Figure 4 shows the average disposition of spare machines into the two unserviceable categories, i.e. awaiting service (W) and being serviced (S).

The servicing conditions, alternative to the primary set, which are worthy of discussion here, are:

(i) Maintaining the same ratio of

$$\frac{\text{average input rate}}{\text{potential output rate}} = \frac{fk}{r} = 0{\cdot}81$$

but halving both the throughput time k and the working capacity r; $(r = 4, k = 3\frac{1}{2})$.

(ii) Halving the ratio $\frac{fk}{r}$ by doubling the working capacity r; $(r = 16, k = 7)$.

(iii) Halving the ratio $\frac{fk}{r}$ by halving the throughput time k; $(r = 8, k = 3\frac{1}{2})$.

Their effects on operational efficiency and other criteria are also illustrated in figures 2–4. Assuming for the purposes of discussion that 98 per cent operational efficiency is a reasonable criterion, it is seen from figure 2 that at this level the three alternative servicing arrangements successively reduce the number of spare machines required to maintain operational efficiency, or alternatively, freedom from emergency. In certain circumstances, the savings in cost may be appreciable; e.g. in the case of machines (aircraft engines) costing some £40,000 each, the effect of halving the throughput time from 7 to $3\frac{1}{2}$ weeks but maintaining the same potential working capacity (alternative iii) thereby reducing the required number of spares from 15 to 7, is to make available up to about £300,000 for the necessary purchase of additional equipment, modification of existing equipment, the engagement of additional labour, or a combination of all three. However, it should be noted that these additional facilities will be largely unused, the utilization factor falling from about 80 per cent to 40 per cent (see figure 3). Nevertheless, if these largely idle resources can be obtained within the cost of the spares saving,

FIGURE 4. *Machines receiving and awaiting service.*

VOL.
5
1954

economic advantage follows, especially so since it may be possible for the additional resources to be diverted to some other productive use when not required for the primary task. It will be seen from figure 4 that the reduction in spares requirements from 15 to 7 arises from two effects: firstly, on the average, only about half the number of machines are being worked on at any one time, although the capacity for working is unaltered; secondly, there is the almost complete abolition of machines waiting for service. In fact, it is seen that with $x = 0{\cdot}41$, the average number of waiting machines does not exceed 1.

In some circumstances, alternative (i) having the same ratio (0·81) of input to potential output could pay dividends, for although the reduction in spare machine holdings is only 3 as compared with the 8 of alternative (iii) just considered, it seems not improbable that the implied concentration of effort could be achieved more by skilled organization and attention to detail rather than by major expenditure on machines or labour. In this event, the utilization factor remains at the higher level of 80 per cent. The reduction in the spare requirements follows only from the reduction in the average number of machines being serviced and, in fact, the average number of machines waiting for service is slightly increased but in any case is less than five.

Summary

The relationships between the number of spare machines, the corresponding probability of an operational emergency, and the relative utilization of the servicing facilities, provide useful criteria for the economic assessment of a spares provisioning programme.

The special circumstances of $f = \frac{r}{k}$, i.e. the maximum rate of output of the servicing facilities being the same as the average rate of unserviceable arisings, are likely to prove uneconomic since:

(i) the rate of increase of operational efficiency is $0\left(\frac{1}{n^2}\right)$ and thus little can be gained by increasing the number of spare machines beyond a certain fairly low limit, and

(ii) as n increases, so does the number of machines waiting for service.

The case of $f < \frac{r}{k}$ is therefore the only practical alternative under which circumstances:

(i) For a given set of servicing parameters, the Law of Diminishing Returns applies to increasing numbers of spare machines and their effect on operational efficiency, but less severely than in the previous case.

(ii) The effect of this Law is minimized by lower ratios of input rate/potential output rate, $f\Big/\frac{r}{k}$.

(iii) Lower ratios $f\Big/\frac{r}{k}$ result in lower utilization of servicing facilities.

(iv) At any level of $f\Big/\frac{r}{k}$, a given operational efficiency is most economically obtained (in terms of spares requirements) by seeking for low compatible values of r and k.

(v) At any level of $f\Big/\frac{r}{k}$, the average number of machines awaiting service does not exceed $\frac{fk}{r-fk}$.

REFERENCE

[1] FELLER, W. (1950), *An Introduction to Probability Theory and Its Application*, vol. 1. John Wiley & Sons.

MANAGEMENT IN THE U.S.A.

A new national society, "The Institute of Management Sciences," has been established with the objective of unifying scientific knowledge that contributes to the understanding and practice of management. The Institute will publish a journal, *Management Science*, which will include research papers and theory of management. Professor C. W. Churchman, of Case Institute of Technology, has been appointed Managing Editor.

Professor W. W. Cooper, Carnegie Institute of Technology, has been elected President of the Institute. Information on membership or subscriptions to the journal can be obtained from Mr. George Kozmetsky, Litton Industries, Beverly Hills, California.

VOL.
5
1954

QUEUEING SYSTEMS WITH PHASE TYPE SERVICE

by

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IN many industries, the activity takes place in each of several successive but distinct phases. For instance, in a typical overhaul procedure, the activity may be conducted in, say, the five stages of stripping, detailed examination, repair, assembly, and testing. A similar situation is met in shops and public offices where parts of the service are given at different counters. This paper considers two types of such a system on the basis of certain assumptions as to the arrival and service time distributions. For the first type, which has particular reference to factory or shop problems, it can be assumed for all practical purposes that the supply of customers is infinite; that is, there are no restrictions to the number of customers who could be present. Secondly, consideration is given to the type of problem in which the number of possible customers is restricted to a finite number, a simple example of such a situation having been discussed in the previous paper in this issue, "The Provisioning of Spare Machines". The present more complex development of the restricted case can also be applied to provisioning problems, while both the restricted and unrestricted models may be applied to certain types of storage problems.

In any of the above situations, knowledge of the probable queue sizes, both initially and inter-stage, together with the numbers actually receiving attention provides quantitative planning criteria. However, the full mathematical treatment of the k-phase problem is complex, and so the method of solution is illustrated for only two phases. Numerical examples of an unrestricted 2-phase system and of a restricted 3-phase system are also given. It is hoped to publish at a later date the general solution for k phases, but meanwhile the 2-phase solution may be directly generalized.

GENERAL ASSUMPTIONS

For convenience, any item requiring service, be it human or mechanical, is referred to as a customer. In both the finite and infinite cases it is assumed that every customer must receive service at each of the service phases in succession. When a customer arrives at an empty first phase, service is immediately started; if it is

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occupied, the customer waits in a queue until it is his turn to be served. This procedure is followed at each successive phase and so inter-phase queuing may take place. Each phase is assumed to have a single server, the service time in the i^{th} phase being exponentially distributed with mean $1/\mu_i$, $i = 1, 2, 3 \ldots$ etc.

The arrival pattern at the first phase is assumed to have two alternative forms:

(i) For the unrestricted case, customers, of which there is a potentially infinite supply, are assumed to arrive at random at a constant average rate λ.

(ii) For the restricted or finite case, it is assumed that customers similarly arrive at random until there is the total number N of potential customers in the service system, there then being no more arrivals until this number drops to $N-1$ or less.

One example of this "Stop Rule" is the case in which the important factor is to keep a specified number of machines operating, unserviceable machines being replaced from a store of maximal size $N-1$. Whenever N machines are unserviceable, it is impossible to operate the specified number of machines and the assumption is that at this juncture operations cease until a serviceable machine becomes available from the servicing system to restore the requisite operational number, e.g. (5).

Mathematical Treatment

In deriving the mathematical solutions to these queueing problems, it is convenient to suppose that the probability of there being, at a time t, n_1 customers in the first phase, n_2 in the second phase, and so on, is denoted by the function $P(n_1, n_2, \ldots, n_k, t)$. The fact of there being n_i customers in the i^{th} phase implies that 1 is actually being served and $n_i - 1$ are waiting in the immediately preceding queue.

Case (i). Two-phase problem. Potential infinity of customers

Using normal methods it can be shown that the following set of equations characterize the system:

For $n_1, n_2 = 0$

$$\frac{\partial}{\partial t} P(o, o, t) = -\lambda P(o, o, t) + \mu_2 P(o, 1, t)$$

For $n_1 = 0$, $n_2 > 0$

$$\frac{\partial}{\partial t} P(o, n_2, t) = -(\lambda + \mu_2) P(o, n_2, t) + \mu_1 P(1, n_2 - 1) + \mu_2 P(o, n_2 + 1, t)$$

VOL.
5
1954

For $n_1 > 0$, $n_2 = 0$

$$\frac{\partial}{\partial t} P(n_1, o, t) = -(\lambda + \mu_1) P(n_1, o, t) + \mu_2 P(n_1, 1, t) + \lambda P(n_1 - 1, o, t)$$

For $n_1, n_2 > 0$

$$\frac{\partial}{\partial t} P(n_1, n_2, t) = -(\lambda + \mu_1 + \mu_2) P(n_1, n_2, t) + \mu_1 P(n_1 + 1, n_2 - 1, t) + \mu_2 P(n_1, n_2 + 1, t) + \lambda P(n_1 - 1, n_2, t)$$

The steady state solutions are obtained by putting the L.H.S. of these equations equal to 0 and are then found to be:

$$P(n_1, n_2) = \left(\frac{\lambda}{\mu_1}\right)^{n_1} \left(\frac{\lambda}{\mu_2}\right)^{n_2} P(o,o). \tag{1}$$

The value of $P(o, o)$ is determined by the normalizing process which requires $\Sigma P(n_1, n_2) = 1$, assuming that $\frac{\lambda}{\mu_1}$ and $\frac{\lambda}{\mu_2} < 1$, and emerges as:

$$P(o,o) = \left(1 - \frac{\lambda}{\mu_1}\right)\left(1 - \frac{\lambda}{\mu_2}\right) \tag{2}$$

It is now convenient to write $x_1 = \frac{\lambda}{\mu_1}$ and $x_2 = \frac{\lambda}{\mu_2}$, so that the several parameters descriptive of the system may be concisely expressed as follows:

Average number of customers in the system as a whole

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + n_2) P(n_1, n_2)$$

$$= P(o,o) \cdot \sum_{n_1=0}^{\infty} x_1^{n_1} \sum_{n_2=0}^{\infty} (n_1 + n_2) x_2^{n_2}$$

$$= P(o,o) \sum_{n_1=0}^{\infty} \left[x_1^{n_1} \left(\frac{n_1}{1 - x_2} + \frac{x_2}{(1 - x_2)^2} \right) \right]$$

$$= (1 - x_1)(1 - x_2) \left[\frac{x_1}{(1 - x_1)^2 (1 - x_2)} + \frac{x_2}{(1 - x_1)(1 - x_2)^2} \right]$$

$$= \frac{x_1}{1 - x_1} + \frac{x_2}{1 - x_2} \tag{3}$$

Further details as to the average disposition of customers in the several parts of the system are similarly obtained and are summarized in Table I.

TABLE I

AVERAGE DISTRIBUTION OF CUSTOMERS THROUGHOUT THE SYSTEM

	Phase 1	Phase 2	Total system
Average number of customers waiting for service	$\frac{x_1^2}{1-x_1}$	$\frac{x_2^2}{1-x_2}$	$\frac{x_1^2}{1-x_1}+\frac{x_2^2}{1-x_2}$
Average number of customers being served	x_1	x_2	x_1+x_2
Average total number of customers	$\frac{x_1}{1-x_1}$	$\frac{x_2}{1-x_2}$	$\frac{x_1}{1-x_1}+\frac{x_2}{1-x_2}$

Finally, the probability distributions for the actual number of customers in any one of the phases are given by:

Probability of there being n customers in Phase 1

$$P_1(n) = \sum_{n_2=0}^{\infty} P(n, n_2) = x_1^n(1-x_1) \tag{4}$$

Probability of there being n customers in Phase 2

$$P_2(n) = \sum_{n_1=0}^{\infty} P(n_1, n) = x_2^n(1-x_2) \tag{5}$$

It will be noted that although the number in any phase would seem to be dependent upon the output from the previous phase, in the steady state each phase settles down in apparent independence.

Case (ii). Two-phase problem. Restricted number of customers

Here it is supposed that, for one reason or another, not more than N customers are available to be in the system. The equations characterizing this case, greater in number than in the unrestricted case because of additional limiting conditions, are:

For $n_1, n_2 = 0$

$$\frac{\partial}{\partial t}P(o,o,t) = -\lambda P(o,o,t) + \mu_2 P(o,1,t)$$

VOL.
5
1954

For $n_1 = 0,\ n_2 = 1, \ldots, N-1$

$$\frac{\partial}{\partial t}P(o,n_2,t) = -(\lambda+\mu_2)\,P(o,n_2,t)+\mu_2 P(o,n_2+1,t) + \mu_1 P(1,n_2-1,t)$$

For $n_1 = 0,\ n_2 = N$

$$\frac{\partial}{\partial t}P(o,N,t) = -\mu_2 P(o,N,t)+\mu_1 P(1,N-1,t)$$

For $n_1 = 1, \ldots, N-1,\ n_2 = 0$

$$\frac{\partial}{\partial t}P(n_1,o,t) = -(\lambda+\mu_1)\,P(n_1,o,t)+\mu_2 P(n_1,1,t) + \lambda P(n_1-1,o,t)$$

For $n_1 = N,\ n_2 = 0$

$$\frac{\partial}{\partial t}P(N,o,t) = -\mu_1 P(N,o,t)+\lambda P(N-1,o,t).$$

For $n_1, n_2 > 0,\ n_1+n_2 < N$

$$\frac{\partial}{\partial t}P(n_1,n_2,t) = -(\lambda+\mu_1+\mu_2)\,P(n_1,n_2,t) + \mu_2 P(n_1,n_2+1,t)+\mu_1 P(n_1+1,n_2-1,t) + \lambda P(n_1-1,n_2,t)$$

For $n_1, n_2 > 0,\ n_1+n_2 = N$

$$\frac{\partial}{\partial t}P(n_1,n_2,t) = -(\mu_1+\mu_2)\,P(n_1,n_2,t) + \mu_1 P(n_1+1,n_2-1,t)+\lambda P(n_1-1,n_2,t)$$

The steady state solutions are similar to those of the unrestricted type of problem and are:

$$P(n_1,n_2) = \left(\frac{\lambda}{\mu_1}\right)^{n_1}\left(\frac{\lambda}{\mu_2}\right)^{n_2} P(o,o) = x_1^{n_1}.x_2^{n_2}.P(o,o) \qquad (6)$$

although the value of $P(o,o)$ is not the same, since it is determined by a summation restricted by the limit $n_1+n_2 \leqslant N$ as follows:

$$1 = \sum_{\substack{n_1=0\\(n_1+n_2\leqslant N)}}^{N}\ \sum_{n_2=0}^{N} P(n_1,n_2) = P(o,o)\sum_{\substack{n_1=0\\(n_1+n_2\leqslant N)}}^{N}\ \sum_{n_2=0}^{N} x_1^{n_1}.x_2^{n_2}$$

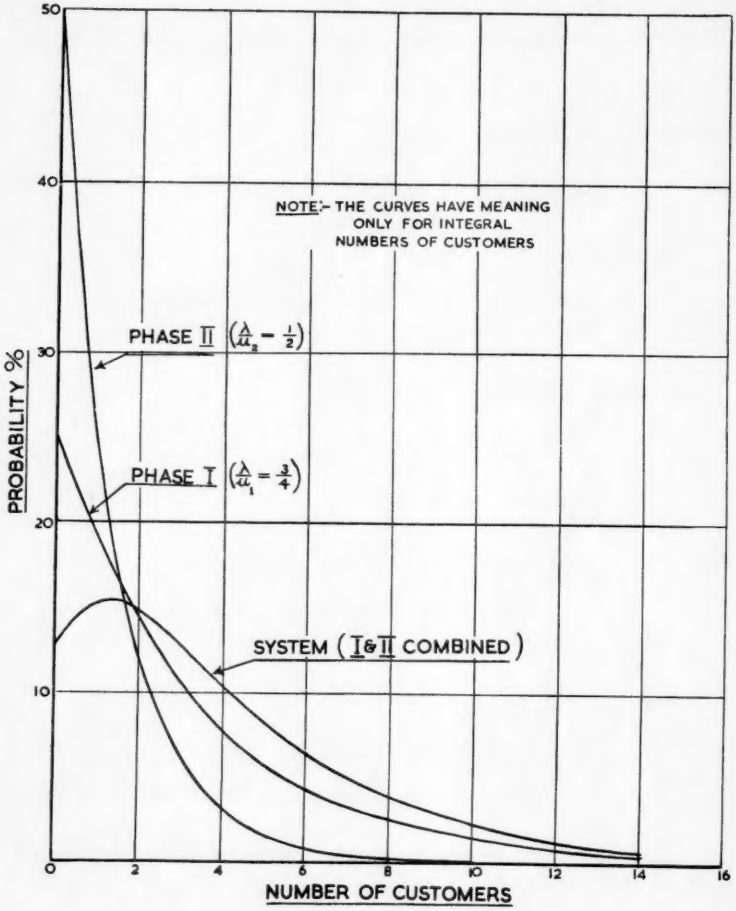

FIGURE 1. *Probability distributions of numbers of customers in the servicing systems (potential infinity of customers).*

The summation, most conveniently effected by the diagonal method, reduces to:

$$\frac{1}{x_1 - x_2} \sum_{v=1}^{N+1} (x_1^v - x_2^v) = \frac{(x_1 - x_2) - (x_1^{N+2} - x_2^{N+2}) + x_1 x_2 (x_1^{N+1} - x_2^{N+1})}{(x_1 - x_2)(1 - x_1)(1 - x_2)}$$

VOL.
5
1954

and so:

$$P(o,o) = \frac{(x_1-x_2)(1-x_1)(1-x_2)}{(x_1-x_2)-(x_1^{N+2}-x_2^{N+2})+x_1x_2(x_1^{N+1}-x_2^{N+1})} \tag{7}$$

As N tends to infinity, this expression tends to $(1-x_1)(1-x_2)$, the value applicable to the unrestricted case [Equation (2)].

In summing the weighted probabilities to obtain the average numbers of customers in various parts of the system, although the series to be summed are finite they lead to rather complicated expressions. Accordingly, in this restricted case, only the formulae for the numbers in the system as a whole are given.

Average number of customers in system

$$= \sum_{\substack{n_1=0 \\ (n_1+n_2\leq N)}}^{N} \sum_{n_2=0}^{N} (n_1+n_2)\,P(n_1,n_2)$$

$$= \frac{1}{x_1-x_2}\left[x_1^2 \sum_{n_1=0}^{N-1}(n_1+1)\,x_1^{n_1} - x_2^2 \sum_{n_2=0}^{N-1}(n_2+1)\,x_2^{n_2}\right] P(o,o)$$

$$= \frac{1}{x_1-x_2}\left[\frac{x_1^2\{1-(N+1)x_1^N+Nx_1^{N+1}\}}{(1-x_1)^2} - \frac{x_2^2\{1-(N+1)x_2^N+Nx_2^{N+1}\}}{(1-x_2)^2}\right] P(o,o) \tag{8}$$

Average number of customers being served:

$$= \sum_1^N P(n_1,o) + \sum_1^N P(o,n_2) + 2\sum_{\substack{n_1=1 \\ (n_1+n_2\leq N)}}^{N} \sum_{n_2=1}^{N} P(n_1,n_2)$$

$$= \left[\sum_1^N x_1^{n_1} + \sum_1^N x_2^{n_2} + \frac{2x_1x_2}{x_1-x_2}\left(\sum_{r=1}^{N-1} x_1^r - \sum_{r=1}^{N-1} x_2^r\right)\right] P(o,o)$$

$$= \left[\frac{(1-x_1^N)x_1}{1-x_1} + \frac{(1-x_2^N)x_2}{1-x_2} + \frac{2x_1x_2}{x_1-x_2}\left\{\frac{x_1(1-x_1^{N-1})}{1-x_1} - \frac{x_2(1-x_2^{N-1})}{1-x_2}\right\}\right] P(o,o)$$

$$= \frac{(x_1+x_2)[(x_1-x_2)-(x_1^{N+1}-x_2^{N+1})+x_1x_2(x_1^N-x_2^N)]}{(1-x_1)(1-x_2)(x_1-x_2)} P(0,0) \tag{9}$$

Average number of customers waiting:

$$= \sum_{2}^{N}(n_1-1)P(n_1,o)+\sum_{2}^{N}(n_2-1)P(o,n_2)+\sum_{2}^{N-1}(n_1-1)P(n_1,1)$$

$$+\sum_{2}^{N-1}(n_2-1)P(1,n_2)+\sum_{\substack{n_1=2\\(n_1+n_2\leqslant N)}}^{N-2}\sum_{n_2=2}^{N-2}(n_1+n_2-2)P(n_1,n_2)$$

$$= \left[x_2^2(1+x_1)\sum_{2}^{N-2}(v-1)x_1^{v-2}+x_1^2(1+x_2)\sum_{2}^{N-2}(v-1)x_2^{v-2}\right.$$

$$\left.+\frac{x_1x_2}{x_1-x_2}\left\{\sum_{2}^{N-1}(v-1)x_1^{v-2}-\sum_{2}^{N-1}(v-1)x_2^{v-2}\right\}\right]P(o,o)$$

$$= \left[\frac{x_2^2(1+x_1)(1-Nx_1^{N-1}+(N-1)x_1^N)}{(1-x_1)^2}\right.$$

$$+\frac{x_1^2(1+x_2)(1-Nx_2^{N-1}+(N-1)x_2^N)}{(1-x_2)^2}$$

$$+\frac{x_1x_2}{x_1-x_2}\left\{\frac{1-(N-1)x_1^{N-2}+(N-2)x_1^{N-1}}{(1-x_1)^2}\right.$$

$$\left.\left.-\frac{1-(N-1)x_2^{N-2}+(N-2)x_2^{N-1}}{(1-x_2)^2}\right\}\right]P(o,o) \qquad (10)$$

Corresponding expressions could be obtained for the average numbers of customers in corresponding states in each phase.

Numerical Examples

Case (i). Two-phase unrestricted model

Inserting the values $\lambda = 1$, $\mu_1 = \frac{4}{3}$, $\mu_2 = 2$, which give $x_1 = \frac{3}{4}$, $x_2 = \frac{1}{2}$ into the expressions in Table I, the following average distribution throughout the system is obtained:

TABLE II

AVERAGE DISTRIBUTION OF CUSTOMERS THROUGHOUT THE SYSTEM

	Phase 1	Phase 2	Total system
Average number of customers waiting for service	2·25	0·5	2·75
Average number of customers being serviced	0·75	0·5	1·25
Average total number of customers	3	1	4

VOL.
5
1954

In this example it will be seen that although the potential servicing rate in phase 2 is 50 per cent higher than that in phase 1, the average number of customers congregated in phase 2 is 67 per cent less than the average number in phase 1 and the number waiting is almost 80 per cent less.

Figure 1 shows the probability distributions of the numbers of customers present in the service system and the individual phases.

Case (ii). Three-phase restricted problem

In certain types of situation, for example spares provisioning, interest is mainly centred on the overall number of customers tied up in the system and not on what is happening in any particular phase. Having regard to the complexities mentioned in connection with equation 3, only the overall picture is considered. Further, the three potential rates of servicing, or the three mean service times, are assumed equal, i.e. $\mu_1 = \mu_2 = \mu_3 = \mu$ (say), and $\frac{\lambda}{\mu} = x$.

Extending equation (6) in accordance with the generalization mentioned in the second paragraph of this paper, the following formula results:

$$P(n_1, n_2, n_3) = x^{n_1+n_2+n_3} P(o, o, o)$$

Writing $n_1+n_2+n_3 = n$, and noting that there are $\frac{(n+1)(n+2)}{2}$ possible ways of obtaining a total of n from combinations of three positive integers:

$$P(n) = \frac{(n+1)(n+2)}{2} x^n P(o, o, o)$$

where $$P(o, o, o) = \left[\sum_{o}^{N} \frac{(n+1)(n+2)}{2} x^n\right]^{-1}$$

$$= [2(1-x)^3/2 - (N+1)(N+2)x^{N+3} + 2(N+1)(N+3)x^{N+2} - (N+2)(N+3)x^{N+1}]$$

The average number of customers somewhere in the servicing system

$$= P(o, o, o) \sum_{o}^{N} \frac{n(n+1)(n+2)}{2} x^n$$

The average number of customers waiting for service

$$= P(o,o,o)\left[3\sum_{1}^{N-1} nx^{n+1} + 3\sum_{1}^{N-2} n(n+1)\,x^{n+2} + \sum_{1}^{N-3} \frac{n(n+1)(n+2)}{2} x^{n+3}\right]$$

The average number of customers being served

$$= P(o,o,o)\left[3x\sum_{o}^{N-1} x^n + 3x^2\sum_{o}^{N-2} (n+1)\,x^n + x^3\sum_{n=o}^{N-3} \frac{(n+1)(n+2)}{2} x^n\right]$$

Although formulae for these summations are easily found, they are rather complicated and are therefore omitted, since the summations themselves are convenient for numerical computation.

The remaining interesting factor is the probability that all the prospective customers have accumulated in the service system. This probability $P(N)$ is immediately calculated from the formulae given in the previous paragraph. As a specific illustration, values of $\lambda = 1$, $\mu = \frac{3}{2}$ give the following tabulated results:

TABLE III

ANALYSIS OF RESTRICTED THREE-PHASE SYSTEM WITH $\frac{\lambda}{\mu} = \frac{2}{3}$

N	$P(N)$	Average number of customers in service system	Average number of customers being served	Average number of customers waiting for service
3	0·343	2·06	1·21	0·46
5	0·193	3·33	1·45	0·92
6	0·146	3·37	1·55	1·66
11	0·037	4·95	1·86	3·03
16	0·009	5·66	1·96	3·68
21	0·002	5·91	1·99	3·91

It will be noted that in this restricted case, the average numbers waiting or being served are not additive to the average total number in the system. However, as N becomes increasingly large the tendency for the phases to behave as if independent as in the unrestricted case, becomes apparent. In fact, for infinite N, the average number

VOL.
5
1954

in the system, being then given by $\frac{3x}{1-x}$ is, for $x = \frac{2}{3}$, equal to 6 to which level the restricted values are seen to tend.

In connection with the problem on spares provisioning discussed in the earlier paper, it is interesting to examine the degree of error arising from assuming an infinite rather than a finite supply of unserviceable items. With a total stock of $N-1$ spares, trouble will be experienced in the operational sphere if the number in the overhaul system exceeds this number. The probability criteria of being free of trouble are:

(1) For the infinite supply of unserviceable items

$$P(n \leqslant N-1) = P(o, o, o) \sum_{o}^{N-1} \frac{(n+1)(n+2)}{2} x^n$$

(2) For the finite supply of unserviceable items

$$P(n \leqslant N-1) = 1 - P(N)$$

Table IV summarizes the two sets of results in terms of the probabilities of accumulating in an overhaul system more than a given number $(N-1)$ of spare items. As might be expected, at any probability level, a larger margin of spares is required in the infinite case, i.e. no stoppage when the spares are all absorbed in the overhaul system, than in the restricted case. This effect emphasizes the need for correct basic decisions as to the type of problem being considered.

TABLE IV

COMPARISON OF INFINITE AND RESTRICTED APPROACHES TO A SPARES PROVISIONING PROBLEM

No. of spare items $(N-1)$	Chance of not exceeding available supply of spares	
	Infinite case	Restricted case
2	0·210	0·657
5	0·532	0·853
10	0·863	0·963
15	0·969	0·991
20	0·993	0·998
∞	1·000	1·000

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VOL
5
195

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ARMY OPERATIONAL RESEARCH IN THE FAR EAST, 1952/54

by

K. PENNYCUICK

ON 31 January, 1952, the writer was instructed to proceed to Singapore to take charge of the existing Operational Research Section, Far East Land Forces and the moribund Operational Research Section, Korea (all the staff there had finished their tours of duty). These two sections had, during their existence, been controlled directly from the War Office. Singapore was reached at the end of April, 1952.

The first thing to be done was clearly to set up an office as far as possible independent of O.R.S., Farelf. That having been done, discussions were immediately held with Director of Operations Malaya and C.-in-C., Farelf as to problems concerning research personnel. It was soon agreed that a study of operations in Malaya was an urgent necessity, and in order to further this study, an operational research section was set up as a part of the Director of Operations' staff. The writer moved his own H.Q. from Singapore to Kuala Lumpur in order that he could devote as much of his time as possible to assisting with the study of the campaign in Malaya.

It was clear at this time also that the study of the operations in Malaya was not sufficient, and accordingly, a Psychological Warfare Research Section was also set up in Kuala Lumpur. This was physically entirely separate from the original section, to be known

VOL.
5
1954

henceforth as O.R.S., Malaya, and this new section, known as O.R.S., P.W., worked with the information services and the police. Details of the work of this section will be given at the appropriate place.

The whole organization was named Operational Research Unit, Far East, and the original O.R.S., Farelf was renamed O.R.S., Singapore. So, by the end of 1952, the organization comprised the H.Q. located at H.Q., Malaya, Kuala Lumpur, and four sections, namely:

O.R.S., Malaya, with Director of Operations' Staff, Kuala Lumpur,
O.R.S., P.W., located with Director General of Information Services (D.G.I.S.) staff, Kuala Lumpur,
O.R.S., Singapore, located at G.H.Q., Farelf,
O.R.S., Korea (which had been revived), located at Rear H.Q., 1, Commonwealth Division.

Apart from the fact that the H.Q. of the organization has now moved back to Singapore, this organization is still in being.* There is no reason, theoretically, why the H.Q. should be located in close proximity to any section. Its location is, in fact, dictated solely by the weight of the work. One region in the Far East which has not appeared under review had no research personnel. This is Hong Kong. It has seemed on many occasions that there is more than sufficient to be done in Hong Kong to justify the presence of a Research Section, but, perhaps, at the moment, Hong Kong is not as important as the other areas, and restrictions on personnel have unfortunately made it impossible to provide a section there.

Operational Research Section, Korea

During the period between the departure of the original O.R.S., Korea and the arrival of replacements, a certain amount of work was done in Korea by a former member of O.R.S., Farelf. This officer had returned to regimental duty and was posted to Korea as a replacement; fortunately, his battalion had not received casualties and he found himself rather at a loose end. Permission was, therefore, given for him to act as a one-man research organization; this was without any further facilities of any sort. He was, however, able to do a certain amount of study in the fields of protective clothing and small arms. For the greater part of the period under review O.R.S., Korea comprised one officer only, and this must be borne

* O.R.S. Korea has ceased to exist since this paper was written.

in mind when considering both the output of work and the way in which he spent his time.

There seems to be a very real difficulty in achieving a thoroughly satisfactory research organization in an area where troops are provided from many countries and are under one overall command. On the face of it, the original organization in Korea where the three British personnel of O.R.S., Korea were attached to O.R.O.,* is the most satisfactory. It means that there is a single research organization only and that administration is reduced to a minimum. There are, however, very serious difficulties. The first of these is that reports published by a research organization are normally distributed through an existing chain of offices, and consequently reports originating in Korea as part of an O.R.O. programme travel to London via Washington with the usual delays caused by clearance at each stage. Whether this difficulty can be overcome in a satisfactory manner remains to be seen, but one way of dealing with it is to authorize a research organization to issue papers on its own authority. This was done in Malaya, and papers issued by Operational Research Unit, Far East, should be regarded as being the considered views of that organization or of its constituent sections only. They do not in the least degree suggest that they are the official views of the military commanders for whom the papers are written, nor do the papers carry any indication as to whether the views expressed in the paper can be, or have been, acted upon. This may, of course, be a disadvantage to the reader who wants to know what has been done about the paper, but it does have the advantage of circulating these papers quite rapidly. The second disadvantage is that specific problems appertaining to the forces of one nation may be very difficult indeed to fit into the programme of an overall research organization. It seemed, when sitting in the War Office, that it was very difficult indeed to get answers from Korea on specific British problems when all research personnel were closely tied in to an American organization; and it may also have appeared to the British forces in Korea, that any questions they wished to put did not receive any answers at all.

A similar problem arose between the Canadian and British forces in 1 Commonwealth division. The Canadians had a small operational research section in Korea. When O.R.S., Korea, was reopened, it was felt, I think rightly, that there was not room for two research organizations in one division, and accordingly, British personnel and Canadian personnel combined quite unofficially,

* i.e., the U.S. Operations Research Office, Far East Branch (O.R.O.; F.E.C.).

VOL.
5
1954

although as far as is known with the full agreement of the relevant authorities. However, this amalgamation later broke down on a change of Canadian personnel, no doubt for reasons similar to the difficulties just outlined. It seems that, so far, we have not really learnt how to organize a Commonwealth research organization, let alone an international one. This problem did not arise in Malaya, where only British troops were involved and an attachment of Australian and American personnel to the Malayan research sections seems to have worked most excellently.

Work done by O.R.S., Korea, during the period falls naturally into two parts. The first part was a quite large-scale study of field defences. The main conclusions reached were that the techniques of building fixed defences such as were developed to a large extent during the 1914–18 War, and which were afterwards very well written up, had been largely forgotten. The study did not produce any startling facts, but it emphasizes the great necessity of proper training in making bunkers and trenches and so on, and even more important, the necessity of ensuring that the principles of such construction are followed in the field. The remaining work of this section was mostly devoted to the study of specific stores and equipment problems. Studies were made on expenditure rates of dry batteries, where, incidentally, it was shown that many batteries reaching the troops in Korea were already at the end of their shelf life. The expenditure rates of such batteries, although in considerable contradiction with staff tables, appeared to be reasonable for the type of warfare under consideration. Expenditure was, in fact, correct, and the estimates of expenditure were wildly wrong. It is noted that, as a result of this study, a considerable speeding up has occurred in the chain of supply and batteries now reaching the troops do so well before the end of their shelf life.

A similar study has since been done on wet batteries, but this was not finished until after the end of the period under review. Other matters studied have been the replacement rate for signals equipment: the problem here is rather interesting. It was observed that the rate of R.E.M.E. repairs to wireless sets in 1 Commonwealth division was increasing very rapidly, and the period between visits to R.E.M.E. by various wireless sets was decreasing rapidly. From an examination of these rates of visits to workshops, estimates were made as to the number of wireless sets (i.e., wireless sets from store) which should be issued each month in order to keep the load on R.E.M.E. fairly constant. It is of interest to note that the actual rate of sets issued from store was appreciably larger than that indicated by the work in the paper.

A considerable amount of thought was put into the problems of armoured vests and steel helmets. A problem of personal protection for the soldier is perennial. The only data which have been collected in this field are statistics on wounds, with and without such protection, and although a study of these data yield much interesting information, it does not yield anything towards a solution of all field problems. It does not, for instance, cast any light whatever on the problem, " Does a man with suitable protection tend to expose himself more than a man without protection?" Nor does it cast much light on the problem of scales of issue of such equipment. The scale of issue for the steel helmet, of course, is one per man, and having been issued with a steel helmet, the soldier leaves it behind him on all possible occasions, particularly when in front of his own lines on a night patrol. He takes the view that the protection afforded by the helmet if he is shot at is not worth the considerably enhanced chance of being shot at if he carries a helmet, for the helmet has three undesirable characteristics. It is very uncomfortable, it makes its wearer rather clumsy and hard of hearing, and it may fall off and disclose its owner's position. Satisfactory techniques of research for dealing with these problems do not seem to exist, and work done is purely by opinion survey—a recognized and reprehensible technique. Later studies by O.R.S., Korea have mostly been in the field of winterization and have been concerned with heating, lighting, and the accidents which are only too frequent as a result of ham-handling of the equipment, and, incidentally, a lack of realization of the fact that petrol in contact with flame burns with some willingness.

Operational Research Section, Malaya

The main work of O.R.S., Malaya in its first year of existence has been a study of operations against Communist terrorists in the Malayan jungle. Clearly, the ideal method of making such a study would be to send a considerable number of research officers into the jungle as observers with instructions to write full accounts of everything they saw and provide detailed answers to certain specific questions—something on the lines of the recording van officers in A.A. Command during the 1939–45 War. In fact, the necessary personnel could not be obtained. It was not even possible to obtain sufficient officers to go and interrogate parties which had been in action against terrorists (one officer, in fact, was borrowed for three months, but during that period did not achieve a sufficient number of interviews to show whether such a process was worthwhile).

Accordingly recourse was made to another technique of the 1939–45 War. Form ZZ was produced, i.e., a detailed action report.

VOL.
5
1954

The original form was a questionnaire for each type of action. Units were asked to make their own copies of the form and send them in to Director of Operations staff after each encounter. This form produced some valuable information, but it did place on the unfortunate soldier a most undesirable burden. It was not satisfactory either from the point of view of the operations analyst, for too much was left to the literary style of the reporter and not sufficient objective questions were asked. Accordingly, a printed form was devised covering all types of operation, and this was filled in compulsorily by all army and police parties in action against terrorists. These completed forms, despite their defects, have proved to be a most valuable mine of information. They have shown a considerable dividend in both tactics and weapon usage, and have also thrown some light on problems of technique in operations such as search theory. The forms have been dealt with in two ways. At four-monthly periods a commentary on the previous four months' forms has been issued; in addition, a number of specific studies have been made on the basis of a year's supply of completed forms. Studies have been made of ambush operations and of patrol operations: of the efficiency of various types of weapons: of techniques of attacking terrorist camps: and of the terrorists' principles in selecting camp sites, and their methods of setting out sentries round such camps.

One of the main weapons in the government's hands against terrorists is that of food control. Generally speaking, it is not possible to live off the land in the Malayan jungle. It is, of course, possible to clear patches of jungle and then grow crops, but unless this is done, the jungle does not provide food. Terrorists living in the jungle, therefore, have to obtain their food either from the villages or towns which they can approach, or from estate lines, or they can make clearings in the jungle and grow their own crops. They used to be able to obtain food supplies from squatters, but as resettlement became more and more complete, this latter source virtually dried up. Control of food from villages is fairly effective, and although it would be idle to pretend there is no smuggling, this is not a major problem; indeed, it is sometimes better to allow food to leave a village and use this as a method of getting contact with the terrorists rather than to stop one specific leak with perhaps the consequence of another unknown leak starting somewhere else. The jungle clearing problem, however, is not easy to resolve. The Malayan climate, humid and hot and with plentiful rain all the year round, except on the East coast, means that plant growth flourishes, and accordingly destruction of crops is not easy. It has been said that the only certain way of destroying a crop in a terrorist clearing is to send a

troop of soldiers into the clearing with instructions to eat the produce! This technique has its drawbacks—many clearings are a considerable distance both in miles and days from the nearest soldiers. Also it is not always possible to spend the necessary time on each clearing in order to ensure that the crop is of no further use to the terrorists. In the case of the deep jungle, spraying of suitable chemicals from the air is an obvious method of crop destruction. If, on the other hand, a soldier can reach the cultivation, hand sprinkling of the crop may be the answer. O.R.S., Malaya, has been studying these problems now for over a year and an officer with suitable botanical qualifications was borrowed from the Royal Air Force, but his efforts have been mainly devoted to carefully controlled experiments on crop destruction with various chemicals. It seems, in passing, to be much easier to prevent a clearing being used again than to seriously affect a crop of well-developed plants.

Later work in O.R.S., Malaya, has been directed to the use of various special devices for ambush purposes. Most of the equipments tried work in the laboratory or under trial conditions but are by no means the sort of thing that any self-respecting person would carry into the jungle. Some recent work has also been done on signals equipment, but this is a field which has not received much attention since 1952.

Operational Research Section, Psychological Warfare

Whether research into psychological warfare and allied topics is a suitable function for an operational research organization has been questioned. In the case of Malaya, there was no other organization which could reasonably take on this responsibility, and there the question was purely academic. In fact, however, the activities of O.R.S., P.W., proved themselves a most satisfactory complement to those of O.R.S., Malaya, and in this case psychological warfare research certainly was a suitable subject.

O.R.S., P.W., has concerned itself mainly with the problem, "Why do Chinese go into the jungle and become terrorists?" (the great majority of all Malayan terrorists are Chinese); and with the problem ,"What is life like in the jungle, and if a man wants to leave the jungle, can he?"; with the problem of the attitude of the civil population to elections, which has a considerable bearing on the attitude to the population to government; and, lastly, with the sheer mechanics of psychological warfare, mainly the various methods of disseminating information. The work on disseminating information is largely studies of the efficiency of voice aircraft, i.e., aircraft fitted with loud-speakers, as a means of communication. Not very

VOL.
5
1954

surprisingly, a slow quiet aircraft is the most desirable machine; rather surprising was the discovery that some of the noises for various Chinese dialects, sounds of an explosive nature, had most undesirable effects on the loudspeaker system, and in an early machine were responsible for the burning out of many loudspeaker coils. It seems to be rather premature to try and make any clear assessment of the value of loudspeaker aircraft, but one or two spectacular performances should be recorded. The first case was the rescue of a Royal Air Force pilot who had parachuted into the jungle from a damaged aircraft; a voice aircraft flew over him, gave him instructions to stay put, and the information that a patrol would come out to rescue him. He was salvaged quite rapidly. The voice aircraft has, on other occasions, shown itself to be a useful method of communicating with troops in the jungle whose wireless sets are not operating. The second spectacular case concerned two terrorists who surrendered. They were invited to record a message to their friends inviting further surrenders, which was duly delivered, and two more of the particular terrorist party emerged from the jungle. On a further occasion, a voice aircraft was flying for the purpose of testing equipment which was not working satisfactorily, and a further surrender was produced from a machine which, on the face of it, was not producing intelligible speech.

Studies on the attitude of the population to elections were still in progress at the beginning of 1954, and it would be improper to discuss them any further in this paper. The problem of "Why does the Chinese go into the jungle?" and what stops him coming out again, however, can be discussed. The method of operation here was to interrogate in great detail all available surrendered terrorists. A special interrogation centre was built in Kuala Lumpur and provided with guards, to keep out unwanted people, not to stop the inhabitants leaving, and a staff of four interrogators was provided by the federal government. One of these interrogators, incidentally, was the well-known novelist Chin Kee Onn. It is true that the sample interrogated, both numerically and statistically, is a bad sample in that it only includes terrorists who have surrendered of their own volition. Nevertheless, certain things are quite clear. The first of these is that the main method of recruiting terrorists into the jungle is to persuade a man to do a small job on behalf of the terrorists, such as smuggling a tin of cigarettes to the jungle's edge, then ask him to do one or two other jobs and then go to him and say, "The police are after you for infringing regulations. You had better come and join us in the jungle." It is not an offence in Malaya to aid terrorists under threat of intimidation; it is, however, an offence not

to disclose that such aid has been given. This method of recruiting, of course, only works satisfactorily in areas where emergency regulations impose severe restrictions on the population. It cannot work at all in those parts of Malaya which have now been declared " white " and it would presumably work most effectively in the so-called "black" areas, where additional communal punishments have been applied. Reading O.R.S., P.W., memoranda on this topic cannot fail to raise doubts as to the wisdom of repressive measures.

On the other side of the picture, coming out of the jungle presents two difficulties. The first is the physical difficulty of leaving the jungle. The would-be ex-terrorist has to know where he is in order to know which road to take out of the jungle, he also has to have an opportunity of getting away from the rest of his group, so the Communist policy is never to leave a man by himself. The other difficulty is that an armed man in the jungle is liable to be shot at sight, and such a man captured is usually hanged. It is most important that the would-be surrenderee should get to the edge of the jungle, hide his weapon, and then report to the police. He will then lead the police back to the weapon and he will then be assured of reasonable treatment. This would appear to be a case where punitive emergency regulations do not discourage people from going into the jungle, but do discourage them from coming out again.

Operational Research Section, Singapore

From the middle of 1952 until the middle of 1953, the main interest of O.R.S., Singapore, lay in small arms trials. The objects of these trials were two-fold. First, to try and get some idea of the efficiency of the soldier with his weapon under shooting conditions made as near those of jungle fighting as possible, and secondly, to carry out specific weapon trials on behalf of the Ministry of Supply. Amongst other things, extensive trials were carried out on the EM2 ·280 rifle, a weapon which was greatly liked and respected by all who saw it, and the Belgian FN rifle, which is to be adopted by the British Army and which received a much more mixed reception. These small arms trials led to a great bulk of information which has since been compared with operational data obtained from O.R.S., Malaya, and has led to the perhaps not unexpected result that for most infantry weapons, the operational factor is about three, i.e., a soldier using the weapon is about a third as effective in battle as on a range. The trials also showed that the carbines such as the Owen, Sten, or the Patchett are only about a third as effective fired from the hip as fired from the shoulder. There is also some reason to believe that however much you train a soldier to use his carbine

VOL.
5
1954

from the shoulder, he has a tendency in times of stress to fire from the hip. The majority of this small arms work has now been transferred to O.R.S., Malaya, which is a more convenient place for dealing with the training aspects which have now arisen; the earlier aspects had to be done in Singapore where range facilities were much better. Towards the end of the period under review, O.R.S. Singapore had been investigating stores problems with particular reference to deterioration of vehicles and use of a vapour phase inhibitor. It has also been interested in the condition of A.A. radar. A very considerable difficulty was being experienced in keeping these sets in running order. It was, however, interesting to note that after O.R.S., Singapore, commenced an investigation into the problem, the difficulties very largely disappeared.

This account has been rather sketchy in its details of the work of the Army Operational Research Sections in the Far East. Many topics have been omitted for security reasons. There is also a certain amount of day-to-day work, and this is not reported or recorded. One matter not so far described is the visit of Dr. J. Cockburn, Mr. H. A. Sargeaunt and Mr. E. C. Williams in September–October 1952. Their mission was basically to see whether there were any fields of study or lines of thought which could be examined in an attempt to further the prosecution of the campaign in Malaya. It would be impudent to express any opinion as to the success of their mission, but it may perhaps be in order to say that their visit to Malaya left behind it a considerable impression of some very large brooms. They removed many cobwebs on thought and produced a great deal of fresh thinking.

Finally, a word on the value of the work done in Malaya. There is no suggestion that the work of the operational research sections has produced any startling improvements. Nevertheless, it is felt that they have made a very worthwhile contribution to long-term defence scientific knowledge.

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OPERATIONAL RESEARCH QUARTERLY

The following is the full price list of the O.R.Q.

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VOL
5
19

ABSTRACTS AND REVIEWS

The Importance of Estimates in Time Studies.
J. P. METMAN.
Tydschrift voor Efficiente Bedrijfsorganisatie, 1954; **9** (4), 116–121. (In Dutch.)

The author points out the shortcomings of the older time study technique which used the measurement of the time per piece or the production per unit of time as the basis of establishing the production rate of the "average workman". An improved technique uses instead the effort, expressed in terms of the product of time and the rapidity of the operator's movements. This effort, corrected for the influence of skill, conditions, consistency, etc., is sometimes called the "activity". Although the production rate is usually assumed to be a linear function of the activity the author shows that this is only approximately true for production rates of from 80 to 115 per cent of the normal. The author concludes by saying that the value of time studies is highly dependent on the skill with which observations are interpreted regardless of the method used.

An Application of Operations Research in the Power Field.
E. W. BOEHNE.
Electrical Engineering, July 1954; **73** (7), 655–658.

The use of operational research is described in the research programme sponsored by the Bonneville Power Administration. The problem concerns the economy loading of this hydro-thermal system while treating the water use of the valley as a whole in order to achieve the greatest possible benefits to all users. The planning of major extensions of the system are being studied as well as the methods of operation.

Two methods of approach to the problem are discussed; the first assumes that the water flow and loads on the system are known in advance in which case the problem can be treated by the calculus of variations; the second is a probabilistic approach in which Monte Carlo methods are applied using fictitious river flows generated on a probability basis from the limited number of recorded historical flows.

The value of operational research is indicated as a management guide, as an operating tool, as a means of evaluating limitations, as a planning tool, as a guide to the importance of establishing accurate flow forecasts and load estimates and as a guide to the location of loads.

VOL.
5
1954

Operational Research—Recollections of Problems Studied, 1940–45.

P. M. S. BLACKETT. (Imperial College.)

Brassey's Annual, 1953.

Professor Blackett provides in this article a personal memoir of some of his experiences in developing operational research groups in the services during the war.

He was first concerned, in 1940, with the problem of the operational use of the new radar sets in Anti-Aircraft Command and he describes three problems which were the main concern of his group. One was the problem of providing a link between good sets and good guns in practice on gun sights, another was the siting of the gun batteries for the A.A. defence of London and the third was the checking of intelligence reports on the number of raiders shot down.

In 1941, he started another group at Coastal Command. Here the problems dealt with were those of the best depth setting for depth charges, the right colour for painting anti-submarine aircraft and the calculation of the probabilities of success of various methods of patrolling a given area.

By 1942, Professor Blackett was called upon to start the Department of Naval Operations Research and was engaged very actively in problems connected with the anti-submarine campaign which culminated in the victories of 1943. These involved considerable controversy about such things as the diversion of heavy bombers from the bombing of Germany to U-boat hunting and the optimum size of convoys. These controversies and the difficulties of persuading people to believe the results of the analysis of operational data are described.

Field Trials of Non-Inflammable Conveyor Belting.

S. L. COOK and B. T. HOULDER. (National Coal Board.)

Since the underground fire at Creswell Colliery (1950) investigations have been made into producing and testing non-inflammable conveyor belting. More recently, controlled trials of non-inflammable belts have been arranged at several collieries.

The objects of the trials were:

1. To determine immediate snags and disadvantages peculiar to the new type of belting.

2. To compare the long term wearing properties of the fireproof belting with that of rubber belting.

The difficulties facing the investigators were increased through the nature of the coal industry, a wide selection of conveyors had

to be used to cover the various conditions encountered in the industry, and as an adequate sample for each type of belting was necessary, the total number of trials arranged became quite large. Supervision of the use of the belting was necessary as was the recording of relevant data: further, this supervision had to be extended to the use of standard belting under similar conditions. Although most of the attributes recorded could be reduced to a figure on an objective scale, some subjective observations were necessary. These were made by the same group of persons for each trial.

Maintenance of Cars.

Tydschrift voor Efficientie en Documentatie, 1953; **23,** 59 (*In Dutch*).

A maintenance system is described involving the use of (a) a maintenance chart listing 58 regular maintenance jobs together with the number of kilometres (or service hours, as the case may be) after which they should be performed; (b) a service chart which together with a works order is issued to the mechanic who is to carry out the items of work called for on the maintenance chart.

It is claimed that this system ensures that every car will be properly maintained with a resultant increase of useful life and a substantial reduction in total maintenance costs.

In addition the system provides the maintenance contractor with a rational basis for the calculation of a fixed fee per car—kilometre (or car-service hour) which for the car owner will result in a substantial saving. Similarly, car fleet owners may rationalize the allocation of funds for maintenance purposes.

The Statistical Design of Experiments.

F. A. A. MENZLER, C.B.E. (London Transport.)

British Transport Review, April, 1954; **3** (1), 49–63.

In attempting to secure improvements in the long wearing properties and reliability of equipment, or in the increase of efficiency and economy in its functioning, transport concerns must often test new methods under service conditions. It is the author's view that these trials must be controlled and conducted scientifically, including the use of statistical methods in the design of the experiment and its subsequent analysis. He stipulates that it is necessary to call in the statistician at an early stage, and planning must be a joint operation between the engineer, operator and statistician. The warning is added that statistical analysis is limited by the initial accuracy of the basic data.

VOL.
5
1954

Two simple examples of service tests are given. In the first the effect on fuel consumption of an engine lubricating oil of low viscosity is considered. This experiment uses a residual variance swollen by bus to bus difference. However, in the second experiment which examines the effect of altering the release pressure of fuel injectors of a bus engine, the injectors are interchanged between busses, and thus the bus effect is removed.

Finally the author describes in detail a controlled experiment conducted under operating conditions. This experiment is again concerned with fuel economy. The effect of different drivers, different vehicles, and the "warming up" trend are examined separately by use of data collected from the same experiment.

La Ricerca Operativa in Gran Bretagna.

MAX DAVIES.

Produttivita, August, 1954, **5** (8), 724–727. (*in Italian.*)

A survey of O.R. in Great Britain since the last war takes examples from work on the importation of iron ore, road traffic analysis and inspection techniques. It touches on recent developments in the U.K. and U.S.A. and suggests some lines of possible usefulness in Italian and other industry.

The Rating of Sinter Plants for Economic Output.

R. F. JENNINGS. (B.I.S.R.A.)

J. Iron & Steel Inst., November, 1953; **175**, 148–256.

From relations between the main design features of sintering equipment and capital and operating costs, a method is presented for calculating the design of a sintering strand to give maximum output at minimum cost per ton from given raw materials. The determination of "economic ratings" is discussed, and the method is applied to a number of plants in current use. It emerges that many of these are not designed on the suggested basis, with the implication that often output is lower than it need be and costs are above the theoretical minimum.

Introducción a los Métodos de la Estadística.

SIXTO RIOS GARCIA.

Instituto de Investigaciones Estadisticas, Madrid.

In two parts, 434 pages.

Determination des Temps Improductifs par des Observations Discontinués.

RENE CHANTAL. (Bureau des Temps Elémentaires.)

Travail et Methodes, September, 1954; 21–27. (In French.)

The "snap-reading" or "ratio-delay" method is described as a tool of operational research. The procedure of the method is explained and its accuracy examined.

Planning Minimum Cost Experiments.

A. DE LA GARZA, D. S. HAWKHURST, L. T. NEWMAN.

U.S. Atomic Energy Non-classified Research and Development Report, K-1017. TID4500.

Assume that the experimental measurement of a certain parameter can be divided into a number of stages, each capable of unlimited repetition, and that the cost and standard deviation of a single measurement at each stage is known. Then for a fixed cost of experimentation there is an optimum division of effort between the stages which maximizes the precision of the parameter estimate. It follows that for a given precision the same optimal division, which is independent of the required precision, gives a minimum cost procedure for obtaining this precision.

Several simple examples are clearly presented and generalization is only a matter of increased algebraic complexity. It would be of interest to know whether the saving on a minimum cost procedure has been found in practice to exceed the cost of estimating the input data and finding the appropriate procedure.

To and Fro: Human Relations in American Industry seen through Dutch Eyes.

ELLA GOUBITZ.

Contractgroep Opvoering Productiviteit, Amsterdam, 1954. *Fl.* 1·75 (*in Dutch*).

The temptation facing any European who goes to the United States on a "productivity mission" under the U.S. Technical Assistance programme must be to try and bring back a suitcase full of useful techniques for immediate distribution to friends and colleagues at home. The importance of resisting this temptation will be appreciated by operational research workers who are conscious of the vast differences between their own industrial situation and that of their opposite numbers in America. Ella Goubitz, a Dutch social scientist

VOL.
5
1954

and industrial consultant in human relations, sets out in this report the reasons why she resisted, and instead tried to bring back from her two years' visit ideas and facts which would help Dutchmen to diagnose and treat their own problems in their own way.

The author went to America armed with a lively appreciation of the cultural differences between that country and her own, but an appreciation which was not distorted by an exaggerated stereotype of American social idiosyncrasies. She is thus able to set out very clearly the difference in the whole basis of management-labour relationships between the United States and Europe which baffles and often angers the undiscriminating American productivity-plugger. In contrast with the American situation, where a fundamental agreement on ends and values underlies a bitter struggle for day-to-day advantage between management and labour, she points out that both sides in Holland accept the need to work together in spite of wide differences on economic and political issues. Her short analysis of this fundamental problem is well worth reading, formulating as it does (by implication) the crucial difference between the approaches of a realist Europe and an idealist America—a difference as important in this field as in foreign policy. Mrs. Goubitz has some interesting remarks to offer, too, on the relative absence of class-consciousness in American industry (is it, incidentally, quite so weak as she implies?), which she suggests may be due in part to a preoccupation with the larger question of race-consciousness. The major part of the credit, however, she thinks is due to the American universities' willingness—in fact their eagerness—to interest themselves in applied science, in inductive method, in operational definition; and in the " dirty end" of industry. The result is a more "democratic" relationship between graduates and others, more delegation of responsibility and more sharing of problems with superiors, equals and subordinates.

From her wide reading and her contacts with about fifty enterprises and institutions (including one trade union) Mrs. Goubitz has extracted a number of important facts which have emerged from the best American research and practice in human relations in industry. She emphasizes one principle which is of particular interest to operational research workers, who are concerned with the resistance of workers (and others) to technical and organizational change: that if decisions are to be effective, then the people who are going to be affected by them must be given a chance to participate in making them. In everyday life this principle might appear to be a matter of common courtesy, but unfortunately industry often seems to take a pride in being an institution set apart from the ordinary processes

of life. Such reports as this, together with the observation and thought which go into their preparation, and the action which (in Holland, at any rate) follows from them, will do a great deal to translate into normal industrial practice the cliché that "workers are human". This particular report will serve also to indicate the very great value of the more carefully-considered aspects of the technical assistance programme. M. W. H.

Operations Research—A Challenge to Modern Management.

GERHARD R. ANDLINGER and others.

Harvard School of Business Administration. 1954, 120 pp.

This is a review of operations research aimed to interest managements in the subject and to encourage them to make more use of it. It is written by a group of nine second year students of the Harvard Graduate School of Business Administration. It suffers inevitably from having many authors whose views cannot always be the same and from lack of personal experience so that in places there is the appearance of uncritical cataloguing of the views of others.

The authors are, however, to be congratulated on a thoughtful and comprehensive review which is designed to answer the questions what is operations research, why is it needed, what has it done and what can it do, how does it do it, who can do it, how should they be organized and what does it cost? The review is written with an enthusiasm which at times leads over rather dangerous ground— for example the main theme, until the last chapter, is astonishment at the slow development of O.R. in business whereas many experienced practitioners are concerned that it is developing too fast for available manpower—but which is nevertheless refreshing. R. T. E.

VACANCY

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VOL.
5
1954